A Stochastic Model of Temporal Variations in Monthly Temperature, Precipitation, Snowfall, and Resulting Snowpack

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Abstract: We report a Monte Carlo representation of the long-term inter-annual variability of monthly snowfall on a detailed (1 km) grid of points throughout the southwest. An extension of the local climate model of the southwestern United States (Stamm and Craig 1992) provides spatially-based estimates of mean and variance of monthly temperature and precipitation. The mean is the expected value from a canonical regression using independent variables that represent controls on climate in this area, including orography. Variance is computed as the standard error of the prediction and provides site-specific measures of (1) natural sources of variation and (2) errors due to limitations of the data and poor distribution of climate stations. Simulation of monthly temperature and precipitation over a sequence of years is achieved by drawing from a bivariate normal distribution. The conditional expectation of precipitation, given temperature in each month, is the basis of a numerical integration of the normal probability distribution of log precipitation below a threshold temperature (3°C) to determine snowfall as a percent of total precipitation. Snowfall predictions are tested at stations for which long-term records are available. At Donner Memorial State Park (elevation 1811 meters) a 34-year simulation matching the length of instrumental record is within 15 percent of observed for mean annual snowfall. We also compute resulting snowpack using a variation of the model of Martinec et al. (1983). This allows additional tests by examining spatial patterns of predicted snowfall and snowpack and their hydrologic implications.

Simulation of seasonal snowpack is an important part of a surface hydrologic model of the southwestern United States. Seasonal snowpack directly affects infiltration and recharge and produces a runoff lag due to water storage. Modeling snowpack during glacial cycles is important because it was the metamorphosis of perennial snowpacks that produced glaciers that occupied many of the higher mountain ranges in the southwest during Quaternary glacial stages. Two-dimensional models of regional surface hydrology are necessary to better understand the system and also to link climate models with proxy evidence of climate change.

Atmospheric General Circulation Models have been used in an attempt to understand Quaternary climate change on a worldwide scale (Kutzbach 1987; COHMAP Members 1988; Street-Perrott 1991), but their resolution is not fine enough for investigation of localized climate change (Kutzbach 1987). For this reason, a local climate model (taking some boundary conditions from global models) has been developed at Kent State University that gives temperature and precipitation based on independent variables available for both the present and last glacial maximum. The model is statistically based instead of physically based, and it can be solved at a resolution as fine as 1 kilometer. The local climate model is simpler and less time consuming computationally than...
global models and may be used to represent uncertainty via a Monte Carlo simulation.

Snow simulation models fall into two categories: energy balance models and index models. Energy balance models rely on mathematical expressions that quantify the exchange of energy between the snowpack and its environment. Index models use one or more parameters as an index of energy exchange. Hoggan et al (1987) and Obled and Rosse (1977) model energy balances within snowpacks. Anderson (1973), Speers et al (1978), Motoyama (1990), and Martinec et al (1983) discuss models wherein surface temperature is the main index of energy exchange across the upper snow surface. We have chosen the methodology of Martinec et al (1983) because it seems most appropriate for this simulation based on the information available from the local climate model.

Methodology

To evaluate spatial and seasonal variations of climate in the southwestern United States, we use a previously reported local climate model that computes monthly temperature and precipitation from a set of independent variables (Stamm and Craig 1992; Stamm 1991). Independent variables are computed from spatial variations of boundary conditions that include terrain elevation, insolation, CO2 concentration, January and July winds, and January and July sea-surface temperatures. Solutions are the product of a canonical regression function, which is calibrated using climate data from 641 stations from six states (AZ, CA, CO, NM, NV, UT) west of 105° west longitude. These data are averaged from 1980-1984 “Summary of the Day” records (U.S. West 1988). Validation of the LCM, using data from 98 climate stations, indicates no significant departures of LCM solutions from climate data (Stamm 1991).

All independent variables can be solved at any point within the calibration domain. For convenience of display here, and as an illustration of the method, we choose to compute climate for a gridded domain that includes the entire drainage basin tributary to Pyramid Lake (Figure 1). The LCM computes five mean canonical variates for each grid point for each year; the LCM also computes standard deviations for each of the five canonical variates (σv).

Figure 2 shows spatial distribution of the standard deviation of the first canonical variate for the solution domain. The standard deviation is greatest in the mountainous areas of the Sierra Nevada and lowest in the northeastern portions of the solution domain, which lie within the Great Basin. This pattern probably reflects the greater natural variability of climate in the mountainous areas as well as the paucity of instrumental records available to parameterize the canonical equations. To represent the uncertainty inherent in the climate model estimates, we make draws from a normal distribution for the first of the five canonical variates for
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Figure 1. Solution domain consisting of the drainage basin tributary to Pyramid Lake, Nevada.

Figure 2. Spatial distribution of the standard deviation of the first canonical variate solved at a 1-km grid cell spacing.

each simulated year. This preserves the correlation structure within and between the months. The five variates are then transformed into monthly temperature and precipitation for use by the snow model.

Temperature predicted by the local climate model (at each grid cell) \( T_m \) is accepted as the mean of a normal distribution, and the log of the predicted precipitation value \( P_m \) is accepted as the mean of a log normal distribution. Variances of temperature \( \sigma^2 \) and log precipitation \( s_p^2 \) are calculated from calibration station data for each month (for a total of 24 values). Snow accumulation is assumed to be the fraction of precipitation that occurs below a critical temperature, \( T_{crit} \) (3°C). We use monthly temperature and precipitation correlation coefficients, again calculated from calibration station data, to construct a bivariate normal distribution (Anderson 1958). We integrate that distribution from \(-\infty\) to \(+\infty\) in log precipitation and from \(-\infty\) to \( T_{crit} \) in temperature to calculate the fraction of precipitation that falls as snow in that grid cell. The integration area is hachured in Figure 3.

The snow ablation routine uses the degree-day factor method of Martinec et al (1983) to compute snowmelt within each of the grid cells. Meltwater depth is calculated from the number of degree-days and a degree-day factor:
Figure 3. Integration of a bivariate normal distribution in temperature and precipitation space to determine the fraction of precipitation that falls as snow.

\[ M = aT \]

(1)

where:

- \( M \) = daily snowmelt depth
- \( T \) = number of degree-days
- \( a \) = degree-day factor = 1.1 \( R \)
- \( R \) = ratio of snow density to water density

We assume a constant degree-day factor of 0.11 cm/°C/day for this study. The monthly snowmelt \( (Q) \) is computed from the snowmelt depth and the ratio \( (S) \) of snow-covered area to total area within each grid cell.

\[ Q = MS \]

(2)

\( S \) varies from zero to one. During the addition of new snow, \( S \) is very close to one, while ablation during the melt season causes increasing patchiness, thus decreasing \( S \). Since the natural variation of \( S \) cannot be computed within our model, it is treated as a random variable drawn from a beta distribution over the range zero to one.
Results

Canonical variates computed by the LCM are randomly perturbed to generate 34 years (matching the length of record) of temperature and precipitation estimates for Donner Memorial State Park (chosen for its illustrious snowfall history; see, for example, McGlashan 1966). The snow model then computes snowfall and snowpack for each month of those 34 years. As Figure 4 shows, the annual cycle dominates the temperature and precipitation signals. There is a factor of two variation in maximum precipitation. The precipitation signal is bimodal (over one year), and that is reflected in the snowfall curve. There is no perennial snowpack for any year within the 34-year solution period. This is not surprising, since these solutions are affected by the 1 km\(^2\) cell size and, therefore, do not capture microtopographic effects that could result in sub-grid-cell size perennial snowpacks.

Figure 4. Temperature (A), precipitation (B), snowfall (C), and snowpack (D) from a 34-year simulation at Donner Memorial State Park, California.
Figures 5 and 6 show the monthly means of modeled snowfall and snowpack plotted versus monthly means of observed snowfall (U.S. West 1988) and snowpack (courtesy of K. Redmond, Western Regional Climate Center). For snowfall (Figure 5), the model over-predicted for November and March and under-predicted for January, February, April, and May. Modeled mean annual snowfall is under-predicted by 14.5 percent. For snowpack (Figure 6), the model under-predicted slightly for January and February and over-predicted for March, April, May, October, and November. March, April, and May snowpack are the most grossly overpredicted.

At present, the snow simulation model assumes a constant density (0.1 g/cm³) for the snowpack, and snowmelt depth is correlated with this density (equation 1). We expect snowpack density to increase during the spring months (Newark et al 1989). Were we to account for this increase, snowpack ablation would also increase, resulting in lower monthly means of modeled snowpack (equation 2) and better fit of modeled and observed snowpack.

The LCM can compute temperature and precipitation for a gridded domain (Stamm and Craig 1992). Solving the snow model at each point of a 1-kilometer grid that includes the entire drainage area of Pyramid Lake (Figure 1) based on the output of the local climate model provides 2-dimensional solutions that illustrate spatial as well as temporal variations in predicted snowpack. Figure 7 shows snowpack extent for April in the second year of a stochastic simulation. Two years are needed to reach a quasi-equilibrium, since the run is started with no extant snowpack. We illustrate solutions for this month because ablation late in the snow season has produced topographically correlated variations in
snowpack extent. The model predicts April snowpack for the Sierra Nevada but not in the lower-lying areas of the solution domain. We expect to extend this model to compute effects of such snowpack on seasonal runoff.

Figure 7. Snowpack extent (in white) for April in the second year of a stochastic simulation solved at a 1-km grid cell spacing.
References


Stamm, JF. 1991. Modeling local paleoclimates and validation in the southwest United States. PhD Dissertation, Kent State University, Kent, OH.


Appendix A
AGENDA

Ninth Annual Pacific Climate (PACLIM) Workshop,
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