SEDIMENT BUDGET
PRINCIPALS AND APPLICATIONS

By

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INTRODUCTION

The framework of sediment budget concepts provides a formalized procedure to account for the various components of sediment flux and the changes of volume that occur within a given region. Sediment budget methodology can be useful in a number of coastal engineering and research applications, including: inferring the amount of onshore sediment transport for a nearshore system that contains an "excess of sediment", determining sediment deficits to downdrift beaches as a result of engineering works at navigational entrances, evaluating the performance of a beach nourishment project, inferring the distribution of longshore sediment transport across the surf zone, etc.

This chapter reviews briefly the governing equations for sediment budget calculations, considers various measurement and other bases for determining the sediment flux components necessary to apply the sediment budget concept and finally for illustration purposes, applies the sediment budget concept to several examples.

GOVERNING EQUATIONS

The governing differential equation for a sediment budget expresses conservation of sediment volume as

\[ \frac{\partial Z}{\partial t} + \nabla \cdot q = S \]  

(1)
in which \( z \) is the vertical coordinate of the bottom, \( \mathbf{q} \) is the sediment transport vector with components \((q_x, q_y)\) and \( S \) represents a source of sand per unit area, see Figure 1. Eq. (1) can also be expressed in terms of the water depth, \( h \), referenced to a fixed datum,

\[
\frac{\partial h}{\partial t} = \mathbf{\nabla} \cdot \mathbf{q} - S
\]

(2)
or in expanded form

\[
\frac{\partial h}{\partial t} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} - S
\]

(3)

**Integrated Form of the Governing Equation**

In some cases, it is possible to apply Eq. (2) directly; however, usually the data available for use in the conservation equation or information required from application of the equation are such that an integrated form is more useful. Integrating Eq. (2) across the beach from \( x_1 \) to \( x_2 \),

\[
\frac{\partial}{\partial t} \int_{x_1}^{x_2} h \, dx = q_x \bigg|_{x_2}^{x_1} - q_x \bigg|_{x_1}^{x_2} + \frac{\partial}{\partial y} \int_{x_1}^{x_2} q_y \, dx - \int_{x_1}^{x_2} S \, dx
\]

(4)
in which \( q_x \bigg|_{x_2} \) and \( q_x \bigg|_{x_1} \) represent the transport per unit width in the offshore direction at the seaward and shoreward ends of the control volume respectively. The first integral term represents the water area, \( A \), between the sand level and the vertical datum, \( z=0 \). If onshore transport occurs at the seaward limit of the control volume, then \( q_x \bigg|_{x_2} < 0 \) and if landward transport of sand occurs due to overwash processes or wind blown sand, then \( q_x \bigg|_{x_1} < 0 \). If fill were added to the profile, then \( S > 0 \). Equation (4) may be useful to apply in this form. However, in some applications, it is helpful to integrate Eq. (4) in an alongshore direction between coordinates \( y_1 \) and \( y_2 \); the result is
Figure 1. Definition Sketch for Sediment Transport Considerations.
\frac{\partial \Psi}{\partial t} = (y_2 - y_1) \left[ q_x \bigg|_{x_1} - q_x \bigg|_{x_2} \right] - Q_y(y_2) - Q_y(y_1) + \frac{\partial \Psi_A}{\partial t} \quad (5)

in which \(Q_y(y_2)\) and \(Q_y(y_1)\) represent the total longshore fluxes of sediment passing through the control volume boundaries at \(y = y_1\) and \(y = y_2\). The quantity \(\Psi\) represents the total sand volume (referenced to some vertical datum) within the control volume and \(\Psi_A\) is the amount of volume added.

Finally, since volume changes are usually observed over some time period \(\Delta t = t_2 - t_1\), in these applications it is appropriate to integrate Eq. (5) over time interval, \(\Delta t\), which yields

\[ \Delta \Psi = (y_2 - y_1) \Delta t \left[ q_x \bigg|_{x_2} - q_x \bigg|_{x_1} \right] - \left[ Q_y(y_2) - Q_y(y_1) \right] \Delta t + \Delta \Psi_A \quad (6) \]

Application

Prior to proceeding further it may be useful to illustrate an immediate application of Eq. (6).

The Nearshore Sediment Transport Study (NSTS) included a field program at Santa Barbara, California to provide data to calibrate the total longshore transport equation,

\[ Q_y = K P_{\xi_S} \quad (7) \]

in which \(P_{\xi_S}\) is the so-called longshore energy flux factor at the wave breaking line and \(Q_y\) is the associated total longshore sediment transport.

The field site including the location of the directional wave gage used in the correlation and the survey lines are presented in Figure 2. In this example, the survey lines extended sufficient distances offshore and upland to
Figure 2. Santa Barbara Survey Plan and Location of $S_{xy}$ Wave Gages.
encompass the entire limit of profile change; thus \( q_x \bigg|_{x_1} = q_x \bigg|_{x_2} = 0 \). Additionally, the navigational channel at the east end of the spit was considered (and believed to be) a complete barrier to longshore sediment transport \( Q_y(y_2) = 0 \) and there were no sediment additions or removals to the system \( (\Delta V_A) = 0 \). Therefore Eq. (6) simplifies to

\[
\Delta V = \bar{Q}_y(y_1) \Delta t
\]

which simply expresses that the change in volume is due to the influx of sediment at an average rate \( \bar{Q}_y(y_1) \), where \( y_1 \) is the location of the wave gage. Thus, the coefficient \( K \) can be determined by combining Eqs. (7) and (8).

\[
\int_{t_1}^{t_2} Q_y \, dt = \int_{t_1}^{t_2} K P_k \, dt
\]

\[
K = \frac{\int_{t_1}^{t_2} Q_y \, dt}{\int_{t}^{t_2} P_k \, dt} = \frac{\Delta V}{\int_{t}^{t_2} P_k \, dt}
\]

such that the numerator is determined from the field surveys and the denominator from the directional wave gage. The results from this study are presented in Table I and Figure 3.

Prior to presenting more examples, it may be useful to review various approaches to augmenting limited data to provide the necessary components in the sediment budget expressions. Because so many useful results can be obtained from equilibrium beach profile concepts, a summary will be presented in the next section.
<table>
<thead>
<tr>
<th>Intersurvey Period</th>
<th>No. of Days</th>
<th>Dredging Event</th>
<th>Total Volume Change (m³)</th>
<th>Net Longshore Component of Wave Energy Flux at Breaking Momentum (N/s)</th>
<th>K = I₂/Pₛ</th>
<th>Net Onshore Flux of Longshore Component of Momentum Sₓᵧ (N/m)</th>
<th>K* = I₂/Sₓᵧ (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Oct. 13, 1979-Nov. 30, 1979</td>
<td>48</td>
<td>No</td>
<td>32,820</td>
<td>85.3</td>
<td>52.2</td>
<td>1.63</td>
<td>27.8</td>
</tr>
<tr>
<td>Dec. 1, 1979-Jan. 20, 1980</td>
<td>31</td>
<td>Yes, Major</td>
<td>65,070</td>
<td>159.1</td>
<td>101.4</td>
<td>1.57</td>
<td>45.4</td>
</tr>
<tr>
<td>Jan. 21, 1980-Feb. 25, 1980</td>
<td>35</td>
<td>Yes, Minor</td>
<td>82,810</td>
<td>295.0</td>
<td>352.4</td>
<td>0.84</td>
<td>119.6</td>
</tr>
<tr>
<td>Apr. 11, 1980-June 3, 1980</td>
<td>53</td>
<td>No</td>
<td>10,290</td>
<td>24.2</td>
<td>76.6</td>
<td>0.32</td>
<td>37.9</td>
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<tr>
<td>June 4, 1980-Aug. 25, 1980</td>
<td>82</td>
<td>No</td>
<td>22,220</td>
<td>33.8</td>
<td>31.7</td>
<td>1.07</td>
<td>17.6</td>
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<tr>
<td>Aug. 26, 1980-Oct. 23, 1980</td>
<td>57</td>
<td>No</td>
<td>38,760</td>
<td>84.8</td>
<td>63.8</td>
<td>1.33</td>
<td>32.6</td>
</tr>
<tr>
<td>Oct. 24, 1980-Dec. 17, 1980</td>
<td>54</td>
<td>Yes, Major</td>
<td>35,640</td>
<td>84.6</td>
<td>64.4</td>
<td>1.31</td>
<td>34.2</td>
</tr>
</tbody>
</table>
Figure 3. Data from Santa Barbara (•) and Rudee Inlet (○) Field Experiments. \( I_L \) vs \( P_{\zeta_S} \), Present and Past Correlations.
EQUILIBRIUM BEACH PROFILES

Introduction

Beach profiles in nature are continuously evolving under the varying action of waves, currents, tides and sediment supply which here will be termed the "forcing functions". If the "forcing functions" were maintained constant, the profile would stabilize into a so-called "equilibrium beach profile", although the equilibration time could be very long. A knowledge of equilibrium beach profiles is useful both in interpreting natural beach conditions and in engineering applications. Problems which can be addressed through equilibrium beach profiles include: beach restoration with a sand of arbitrary size, response of natural or seawalled shorelines to storms and tides, the effects of changes in wave characteristics and thus the seasonal variations in beach profiles, response to sea level rise, and finally through a knowledge of equilibrium beach profiles, it is possible to formulate and test hypotheses on the response of profiles out of equilibrium. One limitation of most presently available equilibrium profile forms is that they are monotonic whereas many profiles in nature are seasonally or perenially barred.

Equilibrium Beach Profile Characteristics

Studies, encompassing several thousand beach profiles from nature and laboratory (with by far a predominance from nature) have demonstrated that most beach profiles can be represented well by the monotonic form

\[ h(x) = Ax^{2/3} \]  

(10)
in which \( h(x) \) is the water depth at a distance, \( x \), offshore and \( A \) is a so-called "scale parameter". It is noted that the parameter \( A \) has dimensions of length to the one-third power (i.e., \( \text{ft}^{1/3} \) or \( \text{m}^{1/3} \)). It can be shown from linear wave theory that Eq. (10) is consistent with uniform wave energy dissipation per unit water volume in the surf zone. Figure 4 shows the origin of one set of field profiles exceeding 500 in number. This set extended from the eastern end of Long Island to the Texas-Mexico border.

Figures 5, 6, 7 and 8 present examples of the fits to various averaged and individual profiles that were used in an assessment to determine the scale parameter. Figure 7 is of special interest in that the sizes of the "sand grains" ranged from 10 to 30 cm in diameter, approximately the size of bowling balls!

Most engineering applications require knowledge of the scale parameter "\( A \)" in Eq. (10). The analyses carried out have shown that \( A \) depends primarily on the sediment size and only secondarily on wave conditions. Figure 9 presents \( A \) vs \( D \) which was the first relationship developed and is recommended if no information is available describing the particular wave height characteristics. It is evident from this figure that beaches composed of larger diameter sediments are steeper, i.e., characterized by larger \( A \) values whereas finer grained beaches are characterized by smaller \( A \) values and thus are milder in slope. The second representation of \( A \) is presented in Figure 10 and includes effects of both sediment size, here represented as the fall velocity, \( w \), and waves, i.e., the breaking wave height, \( H_b \), and wave period \( T \). Examination of Figure 10 will demonstrate the following variation of beach slope with various parameters.
Figure 4. Location Map of the 502 Profiles Used in the Analysis (From Hayden, et al.).
Figure 5. Comparison of Beach Profiles for Data Groups I–V (From Dean, 1977).
Figure 6. Comparison of Beach Profiles for Data Groups VI–X (From Dean, 1977).
Figure 7. Profile P4 (From Zenkovich, 1967). A Boulder Coast in Eastern Kamchatka. Sand Diameter: 150 mm - 300 mm. Least Squares Value of $A = 0.82 \text{ m}^{1/3}$ (From Moore, 1982).

Figure 8. Profile P10 (From Zenkovich, 1967). Near the End of a Spit in Western Black Sea. Whole and Broken Shells. $A = 0.25 \text{ m}^{1/3}$ (From Moore, 1982).
Figure 9. Beach Profile Factor, A, vs Sediment Diameter, D, in Relationship $h = Ax^{2/3}$ (Modified From Moore, 1982).
1.0

S.5

Recommended Relationship

From Hughes' Field Results

From Swart's Laboratory Results

\( h(x) = A x^{2/3} \)

Figure 10. Correlation of Equilibrium Beach Profile Scale Parameter, A, with Combined Sediment/Wave Parameter, \( H_b/wT \).
(2) \( A_B < A_N \)

In this case, with a finer sand placed in the nourishment process than is naturally present on the beaches, the nourished profile will be characterized by a milder slope than the native. The required volume per unit length of beach is

\[
\Psi = \frac{3}{5} \left[ A_N (\Delta x + W_x)^{5/3} - A_B (W_x)^{5/3} \right] + B \Delta x \tag{12}
\]

it is seen that Eq. (A-3) reduces to Eq. (A-2) for the case of \( A_N = A_B \).

(3) \( A_B > A_N \)

In this case, with the placed material being coarser than the native, the two profiles may or may not intersect, depending on the geometry. Thus consideration of two sub-cases is required.

In the first sub-case, the two profiles do not intersect. In this case, the volume required per unit length of beach is the same as for Case 2, in which the profiles do not intersect, i.e.,

\[
\Psi = \frac{3}{5} \left[ A_N (\Delta x + W_x)^{5/3} - A_B (W_x)^{5/3} \right] + B \Delta x \tag{13}
\]

In the second sub-case, the profiles intersect at \( h' \), so sand is only required shoreward of this location, see Figure 11.

The required volume is

\[
\Psi = \frac{3}{5} h'W' + B \Delta x \tag{14}
\]

where \( h' \) is determined by solving the following equation, first for \( W' \)

\[
h' = A_N (\Delta x + W')^{2/3} = A_B W'^{2/3} \tag{15}
\]

which yields

\[
W' = \frac{\Delta x}{\left( \frac{A_B}{A_N} \right)^{2/3} - 1} \tag{16}
\]
Large fall velocities (large diameter sediment)

Steep Slopes

- Small wave heights
- Long wave periods

Small fall velocities (small diameter sediment)

Mild Slopes

- Large wave heights
- Short wave periods

It is noted that all of the above interrelationships are in accord with observations in nature of the variation of wave profiles with wave and sediment characteristics.

Applications of Equilibrium Beach Profiles

Some of the applications of equilibrium beach profiles will be developed below.

Required Beach Nourishment Volumes

This problem must be considered for three separate cases: (1) $A_B = A_N$, (2) $A_B < A_N$ and (3) $A_B > A_N$, where the subscripts "B" and "N" denote "borrow" and "native", respectively and $A$ is the profile scale parameter discussed earlier.

(1) $A_B = A_N$

For this case, the native and nourished profiles would be of the same form. The required volume, $\Psi$, per unit length of beach would be

$$\Psi = \frac{3A}{5} \left[ (\Delta x + W_*)^{5/3} - W_*^{5/3} \right] + B \Delta x$$  \hspace{1cm} (11)

in which $\Delta x$ = shoreline advancement

- $h_*$ = effective depth of limiting motion
- $B$ = berm height
- $W_*$ = width of the nourished surf zone (i.e., out to $h_*$).
a) Sub-Case in which Two Profiles do not Intersect, $h_\star > h'$.

b) Sub-Case in which Two Profiles Intersect, $h' < h_\star$.

Figure 11. Two Sub-Cases of $A_B > A_N'$. 
and \( h' \) can thus be determined as

\[
h' = A_B \left[ \frac{\Delta x}{3/2} \left( \frac{A_B}{A_N} \right)^{-1} \right]
\]

and 
\( h' < h^* \), Profiles do not intersect Eq. (13)
\( h' > h^* \), Profiles intersect Eq. (14)

**Response to Sea Level Rise**

It can be shown that with no additions to or losses of sand from a profile, a rise of sea level, \( S \), will cause a retreat, \( R \), given by the implicit equation,

\[
\frac{R}{W^*} = \frac{S}{B} - \frac{3}{5} \frac{h^*}{B} \left[ 1 - \left(1 - \frac{R}{W^*} \right)^{5/3} \right]
\]

(18)

It can be shown that for retreat magnitudes, \( R \), which are small compared to the surf zone width, \( W^* \), Eq. (18) simplifies to

\[
R = S \frac{W^*}{h^* + B}
\]

(19)

which is recognized as the so-called "Bruun Rule" presented by Bruun in 1962 to represent this phenomenon.

**Volume of Sediment Transported Offshore to Various Depths Due to Sea Level Rise**

The case just considered results in an offshore transport of sediment due to sea level rise. However, if there are no longshore gradients of sediment transport, there is no loss of sediment along a profile and accurate
consecutive surveys encompassing the entire region of profile change referenced to the same vertical datum should result in the same total volume.

Of relevance to the present study is the case in which the profiles do not extend a sufficient distance offshore to encompass the entire region of profile change. For example, if the surveys extended only out to the profile intersection point, the apparent volumetric loss would be the hatched area above the intersection point in Figure 12.

It can be shown that the non-dimensional apparent volume loss is approximately

\[
\frac{V_E}{W_*S} = \frac{(B/h_* + h_S/h_*)}{(B/h_* + 1)-\left(\frac{h_S}{h_*}\right)^{3/2}}
\]

in which \(h_S\) is the offshore depths to which the surveys are conducted. The profile intersection depth, \(h_I\), referenced to the datum before the sea level rise is approximately

\[
\frac{h_I}{h_*} = \frac{4}{9} \frac{1}{(1 + B/h_*)^2}
\]

In Eq. (20), the quantity \(W_*S\) is the nominal amount usually referenced as the volumetric erosion due to sea level rise.

To examine Eq. (20) further, Figure 13 presents the non-dimensional apparent volumetric erosion \(V_E/(W_*S)\) vs \(h_S/h_*\) for ratios \(B/h_*\) of 0 and 0.25. It is seen that the greatest volume of apparent erosion possible (and only if the surveys were carried out precisely to the intersection point) are 15% and 20% of the nominal value respectively for \(B/h_*\) values of 0 and 0.25. The reason for this can be determined by examining the effects of shifting a profile vertically upward (due to sea level rise) and landward (to conserve
Figure 12. Definition Sketch Showing Portions of Profile over which Erosion and Deposition Occur Due to a Landward and Upward Profile Translation.
Figure 13. Relationship of Non-Dimensional Apparent Eroded Volume to Non-Dimensional Survey Depth.
sediment). The apparent total erosion due to a vertical displacement $S$ is clearly $W*S$; however, when the profile is also shifted landward not only is the net erosion reduced (indeed to zero), but the local erosion (landward of the limit of offshore motion) is reduced substantially relative to $W*S$.

Additional Applications of Equilibrium Beach Profiles

Additional applications of equilibrium beach profiles that will only be mentioned here include: the response of natural and seawalled profiles to storms and sea level rise and providing a basis for examining the transient response of profiles, i.e., profiles that are not in equilibrium. The reader is referred to Dean (1983) for additional information.

ADDITIONAL USEFUL APPROXIMATIONS IN SEDIMENT BUDGET CALCULATIONS

This section presents a number of useful approximations and aids in supplementing limited data in order to carry out sediment budget calculations.

Volumetric Changes Associated with Shoreline Changes

In many cases, there may be data available for shoreline changes, but not volumetric changes. If the profile remains unchanged as the profile advances, the associated volume change, per unit length of beachfront, $\Delta V$, is

$$\Delta V = (h_0 + B) \Delta x \quad (22)$$

such that an advancement (retreat) of the beach would be associated with a gain (loss) of volume. In the case in which an entire barrier island is advancing (retreating) without change of form, then the change of volume is (Figure 13)
\[ \Delta V = (h^*_o - h^*_b) \Delta x \]  

in which the subscripts "o" and "b" denote ocean and bay, respectively.

APPLICATIONS

One application of the sediment budget concept was presented previously. This section presents several additional specific examples.

South Shore of Long Island

The south shore of Long Island is 134 km in length and due to the impact of a number of major storms, several surveys have been conducted since 1933, although the quality of the more recent data is much better than the earlier data.

Briefly, referring to Figure 14, the net transport along the shoreline is from east to west and shoreline surveys indicate that there is not sufficient erosion to provide the quantity of sediment transport documented at Fire Island Inlet (approximately 350,000 m³/yr). This strongly suggests an onshore transport of sediment of considerable magnitude. Two time periods of reasonably high quality are available: 1940-1955 and 1955-1979. The onshore sediment transport, \( q_x(x_2) \), is inferred from Eq. (6) as

\[ - \bar{q}_x \bigg|_{x_2} (y_2 - y_1) = \frac{1}{(t_2 - t_1)} \left[ - \bar{q}_x \bigg|_{x_1} (y_2 - y_1) + Q_y \bigg|_{y_1} - Q_y \bigg|_{y_2} - \Delta W_A \right] \]  

Applying the above equation to the data presented in Table II from the two time periods results in a substantial variation of the onshore sediment transport, i.e.
Figure 14. The South Shore of Long Island and the Questions of the Magnitude of Net Onshore Sediment Transport.
Table II

SUMMARY OF SEDIMENT BUDGET ANALYSIS
MONTAUK POINT TO FIRE ISLAND INLET

<table>
<thead>
<tr>
<th>Time Span</th>
<th>Number of Years</th>
<th>Net Change (yd³/yr)</th>
<th>Fill Additions (yd³/yr)</th>
<th>Washover Aeolian Transport (yd³/yr)</th>
<th>Allowance For Sea Level Rise (yd³/yr)</th>
<th>Transport Past Democrat Point (yd³/yr)</th>
<th>Inferred Transport Onshore (yd³/yr)</th>
</tr>
</thead>
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<tr>
<td>1940-1955</td>
<td>15.4</td>
<td>1,356,929</td>
<td>313,032</td>
<td>94,330</td>
<td>64,606</td>
<td>0</td>
<td>1,359,349</td>
</tr>
<tr>
<td>1955-1979</td>
<td>24.5</td>
<td>7,501</td>
<td>420,444</td>
<td>32,825</td>
<td>64,606</td>
<td>400,000</td>
<td>294,710</td>
</tr>
<tr>
<td>1940-1979</td>
<td>39.9</td>
<td>528,333</td>
<td>378,987</td>
<td>56,564</td>
<td>64,606</td>
<td>245,614</td>
<td>705,622</td>
</tr>
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Inferred Onshore Transport Rate (Column (8)) = Column (3) - 0.5 x Column (4) + Column (5) + Column (6) + Column (7).
### Inferred Onshore Time Period Sediment Transport

<table>
<thead>
<tr>
<th>Time Period</th>
<th>Inferred Onshore Sediment Transport</th>
</tr>
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<tr>
<td>1940-1955</td>
<td>1,040,000 m³/yr</td>
</tr>
<tr>
<td>1955-1979</td>
<td>225,000 m³/yr</td>
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Based on these results, it was concluded that the onshore sediment transport was episodic and possibly the result of infrequent storm conditions or highly varying storm seasons. Also, the representative annual onshore sediment transport, based on a weighted average of the above two values is 540,000 m³.

In applying and interpreting the sediment budget approach, it is always useful to question the reasonableness of the results and the inferred large magnitudes of sediment transport at Long Island is no exception. The continental shelf off Long Island is a glacial outwash plain composed of poorly sorted sediments. Comparison of a representative profile with two equilibrium profiles for sediment of the appropriate size (Figure 15) indicates that indeed the profile is probably "out of equilibrium" with an excess of sediment which would tend to result in onshore sediment transport due to the milder bottom slopes (compared to equilibrium). Moreover, the size of the sediment is such that it tends to be transported only by the larger waves associated with infrequent storms.

**Brevard County, Florida**

This example provides a good case study illustrating the application of a sediment budget analysis to determine the effect of a channel entrance. Port Canaveral entrance was cut in 1951 and is therefore a relatively young entrance. Figure 16 presents a location map for this entrance on the east
Figure 15. Comparison of Actual and Idealized Profiles. Actual Profile Approximately Midway between Sinnecock and Moriches Inlets.
Figure 16. Location Map of Brevard County and Port Canaveral Entrance.
coast of Florida. Shoreline change results are available for this area commencing in 1855. Table III presents a chronology of events relevant to changes in the shoreline.

**TABLE III**

<table>
<thead>
<tr>
<th>CHRONOLOGY OF SIGNIFICANT EVENTS AT PORT CANAVERAL ENTRANCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Entrance Cut</td>
</tr>
<tr>
<td>Jetties Constructed</td>
</tr>
<tr>
<td>Beach Nourishment Project</td>
</tr>
</tbody>
</table>

The longshore sediment transport is predominantly toward the south at rates estimated up to 270,000 m$^3$/yr although there is substantial uncertainty in this estimate. The shoreline changes following construction of the inlet coupled with a sediment budget analysis provide a basis for improving this estimate.

Figure 17a presents the pre-entrance shoreline change rates. The abscissa represents distance toward the south with the total length shown representing a distance in excess of 66 km. The ordinate represents shoreline change rate. It is seen that in the time period 1877-1951, which is dominantly pre-entrance, the shoreline was accreting over more than 80% of the shoreline. The average shoreline change rate over this 78 year time period is approximately 0.3 m/yr. During the 19 year period (1955-1974) subsequent to the cutting of the entrance, erosion had commenced with the maximum erosion existing immediately downdrift (south) of the entrance. The maximum erosion rate was approximately 5 m/yr over this 19 year period and the effect extended some 5.5 km south of the entrance.
a) Effects of Channel Entrance on Down Drift Beach Stability

b) Shoreline Changes Following 1974 Nourishment Project

Figure 17. Effects of Establishment of Cape Canaveral Entrance and Subsequent Nourishment Project on Downdrift Beaches.
In order to apply a sediment budget approach to determine the longshore transport rate, it would be desirable to have available volumetric changes downdrift of the entrance. However, lacking volumetric data, it is possible to utilize Eq. (22) which relates volumetric change to plan area change. Integrating the plan area in Figure 17b to determine the plan area rate of change between the pre-entrance and post-entrance rate of change yields approximately 15,000 m$^2$/yr. Considering a vertical dimension of 8 m over which the profile is shifted landward in the erosion process with the form remaining unchanged and making the assumption (probably quite true) that there is no cross-shore transport outside the shoreward or seaward limits accounted for here, Eq. (6) simplifies to

$$\frac{\Delta H}{\Delta t} = Q_y(y_1) - Q_y(y_2)$$

(25)

in which $Q_y(y_1)$ and $Q_y(y_2)$ represent the longshore transport at the jetty and that outside the region affected by the entrance. The quantity $Q_y(y_2)$ of course is the unaffected net longshore sediment transport and is the transport of interest. If it is assumed that the south jetty is impermeable, then $Q_y(y_1) = 0$, and

$$Q_y(y_2) = \frac{\Delta H}{\Delta t} = 15,000 \text{ m}^2/\text{yr} (8 \text{ m})$$

$$= 120,000 \text{ m}^3/\text{yr}$$

It is of interest to examine the effect of the south jetty if it were not impermeable. It is clear that if the jetty is permeable, that sand would be lost into the entrance through the jetty, i.e. $Q_y(y_1 < 0)$, and
\[ Q_y(y_2) = - \frac{\Delta V}{\Delta t} - Q_y(y_1) \] (26)

Thus a leaky jetty would tend to overemphasize the amount of longshore sediment transport if it were assumed to be sand tight.

Ideally one would have available other data indicating whether or not the permeability of a jetty could result in significant transport through the jetty. In particular the orientation of the shoreline relative to the jetty can serve as a qualitative indication of permeability. For example, if the shoreline is aligned perpendicular to the jetty, there is probably little sediment transport through the jetty. If the shoreline forms an acute angle with the jetty, the jetty is clearly "leaky" and significant transport over or through the jetty is probably occurring. In the case of Port Canaveral Entrance, inspection of the south jetty demonstrates that it is "leaky"; however, the shoreline orientation is such that this transport component is not considered to be significant.

Having explored the net longshore sediment transport through analysis of the erosional phase of the shoreline south of Port Canaveral Entrance, it is useful to examine the shoreline changes subsequent to a relatively substantial nourishment project. In 1974, a nourishment project consisting of approximately 2 million cubic meters was placed along a 3,400 m segment of shoreline immediately south of the south jetty. The shoreline changes from 1972 to 1986 are presented in Figure 17b. This figure is based on an entirely different data source and over a completely different time period, yet it is of interest to note that outside the region of influence of the entrance the shoreline change rates are very similar to those based on the period 1877–1951. This figure clearly shows qualitatively that the erosive "wave" has now
moved some 27 km south of the entrance and that it is being followed by an accretional wave resulting from the 1974 beach nourishment project. It is of interest to attempt to separate the erosional and accretional waves. At present, this can only be accomplished in an approximately manner.

Ocean City Inlet, Maryland

Ocean City Inlet was caused by a hurricane in 1933. Within the next few years, jetties were constructed to maintain the channel navigable. Major changes that occurred in the vicinity included impoundment at the north jetty, severe erosion of the shoreline south of Ocean City Inlet and the development of a substantial shoal offshore and slightly south of the entrance centerline. Additionally, substantial erosion of the ocean shoreline occurred with landward migration. Figure 18 presents map showing the location of Ocean City Inlet.

This example describes a field effort to explain the cause of erosion to the northern portion of Assateague Island and also presents the results of an attempt to compute a sand budget.

The sand budget components were determined by a combination of field and computational procedures. The net longshore sediment transport was based on a combination of impoundment measurements against the north jetty after it was constructed and wave observations. The computational results are presented in Figure 19 and are to be compared to the impoundment results against the updrift jetty of 120,000 m$^3$/yr. The computed values based on wave observations exceed the impoundment values by a factor of 6, however, the observations were made at a distance of 13 km south of the inlet and it is known that transport increases to the south. The volumes lost and gained by migration of the barrier island were based on Eq. (23) and even though the
Figure 18. Location of Ocean City Inlet and Influence of Bathymetry in Reducing Wave Action Along Northern Segment of Shoreline Cape Henlopen to Fishing Point.
Figure 19. Averages and Ranges of Net Longshore Transport, Assateague Island, Based on LEO Data for the Years 1973, 1974 and 1975.
island basically retained the original width, the volumes lost exceeded those gained by 38,000 m³/yr due to \( h_{o*} > h_{b*} \) of Eq. (23).

Field observations of the south jetty demonstrated that it was both low and porous allowing considerable sediment to flow from north Assateague Island into the entrance channel from where it was jetted to partially account for the growth of the offshore shoal. Field measurements were conducted with a "swash trap" constructed of reinforcing rod and porous plastic mesh, see Figure 20. Deployment of this trap quantified the flux of sediment through the south jetty and these results extrapolated to account for the variable wave and tide conditions resulted in an estimated flux of 31,000 m³/yr over and through the south jetty. As support for the concept of the low and permeable south jetty "draining" sand off Assateague Island's north beach, beach profiles showed a trend of decreasing berm height from the natural elevation of 2.4 m MSL—several thousand meters south of the south jetty to the 1.4 m crest elevation of the south jetty, see Figure 21.

Finally, Figure 22 presents a best attempt to account for all the components of the sediment budget. It is seen that there is a residual or "mismatch" of 38,000 m³/yr out of a total of 680,000 m³/yr, or a discrepancy of about 11%.

Cross-Shore Distribution of Longshore Sediment Transport

A knowledge of the cross-shore distribution of longshore sediment transport is important in a number of engineering applications, including weir jetty design. Several methods have been explored for inferring the cross-shore distribution of longshore sediment transport, including tracers, local traps and in situ point measurements of suspended sediment, longshore currents and bed load traps.
Figure 20. Sediment Trap Used in Measurements of Sand Transported by Wave Swash.
Figure 21. Comparison of Elevations of South Jetty and Beach Profiles at Two Locations South of Jetty.
Figure 22. Results of Sediment Budget Analysis.
The method described here is based on Eq. (3), rewritten below
\[
\frac{\partial h}{\partial t} = \frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y}
\]
in which the source term, S, has been set equal to zero. Consider a barrier placed instantaneously across the surf zone such that at the barrier, \( q_y = 0 \). Also, consider that profile measurements are conducted before the near-barrier profiles steepen to the degree that cross-shore transport is induced. Thus if the profile is initially in equilibrium and remains so, the cross-shore transport \( Q_x = 0 \).

Integrating Eq. (3) along a contour from the barrier to a location, \( y_2 \), unaffected by the presence of the barrier,
\[
q_y(y_2, h) = q_y(y_1, h) + \frac{1}{\Delta t} \int_{y_1}^{y_2} \Delta h(h) dy
\]

Fulford (1975) has applied this method in the laboratory. Bodge (1986) has applied this method in the laboratory and field and has presented a method to remove the first-order time-varying effects of the tide. Figure 23 presents an example from Bodge of the cross-shore distribution for plunging waves.

**Entrance to St. Andrews Bay, Florida**

This situation is somewhat similar to Brevard County, Florida. The entrance was cut across a barrier island in 1934 and thus in geological time scale is a fairly young inlet.

Shoreline change data are available for the period 1855–1934 as shown in Figure 24a. There were areas of erosion and accretion; however, for the 36 km section presented in Figure 24, the average shoreline change was one of
Figure 23. Examples of the Cross-Shore Distribution of Longshore Sediment Transport (Bodge, 1986)
a) Shoreline Change Rates Prior to Cutting Entrance to St. Andrews Bay, 1855-1934 (79 Years).

b) Comparison of Shoreline Change Rates Prior to Cutting Entrance to St. Andrews Bay, 1855-1934 (79 Years) and Subsequent to Cutting Entrance, 1934-1984 (50 Years).

Figure 24. Effect of Cutting Entrance to St. Andrews Bay in 1934 on Downdrift Shoreline.
accretion, averaging approximately 0.3 m/yr. The average shoreline change for the period 1934 to 1984 is presented in Figure 24b. It is seen that immediately downdrift (west) of the entrance, the shoreline change rate had been altered from one of accretion of approximately 1 m/yr to one of erosion in excess of 2.5 m/yr, i.e. a differential erosion in excess of 3.5 m/yr. In this 50 year period, the shoreline immediately downdrift of the inlet eroded by more than 125 m whereas under natural conditions, the projected accretion would have been approximately 50 m.

In the case of St. Andrews Bay Entrance, there were three contributing factors to the downdrift erosion:

1. Approximately 7.5 million cubic meters of sand was dredged from the entrance channel and spoiled in deep water,
2. After the inlet was cut, the ebb tidal shoal developed accumulating approximately 3,000,000 cubic meters, and
3. The jetties were extremely leaky which contributed to the necessary dredging in (1), above.

As a verification that the leaky jetties contributed to the required shoaling (and thus the downdrift erosion), the net longshore sediment transport is estimated as 60,000 cubic meters per year whereas the downdrift erosion was in excess of 160,000 cubic meters/yr.

Offshore bathymetry is insufficient to estimate the downdrift volumetric erosion rate. However, as before, it is possible to estimate this based on planform changes and Eq. (22). The average annual differential planform changes are 20,300 m² erosion which when combined with a profile change of 8 m, yields an annual volumetric erosion rate of 160,000 cubic meters or a total erosion over the 50 year period of 8,100,000 cubic meters.
Rudee Inlet, Virginia

Rudee Inlet, Virginia was a second field location in the Nearshore Sediment Transport Study program where transport rates were studied to investigate the longshore sediment transport equation. The net longshore sediment transport is toward the north. The south jetty of the inlet includes a weir section which allows sediment to enter a deposition basin from which the sediment is dredged and transported by pipeline across the entrance to Virginia Beach, see Figure 25.

When the experiment was planned, it was assumed that the weir allowed the net longshore transport to pass over the weir during transport toward the north and that sediment transported during reversals was relatively small. Fortunately, the survey plan included a substantial portion of the updrift beach as shown in Figure 26.

It was found that the net longshore sediment transport toward the north was a small fraction of the gross transport. During periods of northerly transport, relatively large volumes of sediment are transported and deposited updrift of the north jetty whereas only a small quantity enters the deposition jetty. During periods of sediment reversal, the volumes stored updrift of the jetty is diminished, but sand continues to be carried across the weir section.

Thus in order to obtain the appropriate volume for correlation with the longshore wave energy flux factor, it is necessary to include the volume accumulated in the deposition basin and that either deposited or eroded from the updrift (south) beach during the intersurvey period.

SUMMARY

The formalized framework provided by a sediment budget analysis is useful in many general and specific coastal engineering applications, including
Figure 25. Rudee Inlet, Showing Weir Jetty and Impoundment Basin (Adapted From Needham and Johnson, 1972).
Figure 26. Rudee Inlet Survey Plan and Location of $S_{xy}$ Wave Gage.
interpreting natural and altered systems, inferring onshore (or offshore) sediment transport for a system that is out of equilibrium, determining sediment transport rates from volumetric measurements, estimating the cross-shore distribution of longshore sediment transport, and many others. This framework should be developed and applied by all practicing coastal engineers confronted by the difficult problems of understanding a system, many times with inadequate data available.

REFERENCES

