SHORELINE EROSION DUE TO EXTREME STORMS AND SEA LEVEL RISE

By

R. G. Dean

December 1, 1983
A summary is presented of research conducted on beach erosion associated with extreme storms and sea level rise. These results were developed by the author and graduate students under sponsorship of the University of Delaware Sea Grant Program.

Various shoreline response problems of engineering interest are examined. The basis for the approach is a monotonic equilibrium profile of the form $h = Ax^{2/3}$ in which $h$ is water depth at a distance $x$ from the shoreline and $A$ is a scale parameter depending primarily on sediment characteristics and secondarily on wave characteristics. This form is shown to be consistent with uniform wave energy dissipation per unit volume. The dependency of $A$ on sediment size is quantified through laboratory and field data. Quasi-static beach response is examined to represent the effect of sea level rise. Cases considered include natural and sea-walled profiles.

To represent response to storms of realistic durations, a model is proposed in which the offshore transport is proportional to the "excess" energy dissipation per unit volume. The single rate constant in this model was evaluated based on large scale wave tank tests and confirmed with Hurricane Eloise pre- and post-storm surveys. It is shown that most hurricanes only cause 10% to 25% of the erosion potential associated with the peak storm tide and wave conditions. Additional applications include profile response employing a fairly realistic breaking model in which longshore bars are formed and long-term (500 years) Monte Carlo simulation including the contributions due to sea level rise and random storm occurrences.
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By

R. G. Dean

Summary of Research Results Developed Under
University of Delaware Sea Grant Project R/T-24

December 1, 1983

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NOTATION

A, A' Scale parameter in equilibrium beach profile expression, see Eq. (10)
B Berm height
D Sediment particle diameter
D Wave energy dissipation rate per unit volume
Dk Equilibrium wave energy dissipation rate per unit volume
F Fall velocity parameter, also "function of"
g Gravitational constant
Hb Breaking wave height
Ho Deep water wave height
h Water depth
hb Breaking depth
h1, h2 Depth dimensions to features of perched beach, see Figure 32
hb2 Breaking depth under storm surge conditions
hw1 Water depth at toe of seawall under normal conditions
K, K* Rate constants for offshore sediment transport
K' Longshore sediment transport proportionality factor
kD Constant defined by Eq. (17)
L0 Deep water wave length
L1, L2 Positions of contours L1 and L2
m Shape parameter in equilibrium beach profile, see Eq. (7)
P£s Longshore component of wave energy flux at breaking
p Barometric pressure
Qs Offshore sediment transport rate per unit width
R Beach recession
Rmax Radius to maximum winds in a hurricane
S Water level increase, such as storm tide
T Wave or storm tide period
t Time
V F Translational speed of a hurricane system
W Equilibrium distance between two beach profile contours, also barrier island width, see Figure 35
W2 Breaking zone width under storm surge conditions
\( w \) Sediment fall velocity

\( y_F \) Landfall location of a hurricane center

\( x \) Offshore coordinate

\( x_n \) Offshore coordinate to \( n^{th} \) contour

\( x_1, x_2 \) Horizontal dimensions to features of perched beach, see Figure 32

\( \alpha \) Rate constant

\( \beta \) Translation direction of hurricane system

\( \kappa \) Proportionality factor in spilling breaker model

\( \sigma \) Angular frequency (\( \equiv 2\pi/T \))

\( \theta \) Average vertical angle over active portion of beach profile

\( \rho \) Mass density of water

\( \gamma \) Specific weight of water (\( \equiv pg \))

\( \pi \) Numerical constant = 3.14159....

\( \eta \) Storm tide
I. INTRODUCTION

The purpose of this report is to summarize research results conducted under University of Delaware Sea Grant sponsorship of Project R/T-24 "Shoreline Erosion to Extreme Storms and Sea Level Rise".

The motivation for this project arose from the recognition of the need for difficult future decisions with inadequate data/knowledge relative to shoreline erosion, shoreline development consequences and/or remedial erosion measures. Much of the shoreline has been developed with the construction of expensive and substantial upland structures. The attraction for the placement of these structures included the beauty and recreational advantages of the beaches. In an era of gradual sea level rise and associated inexorable erosional trend, the beaches recede at an average rate of 30 cm to 1 m per year. Single storm events can cause dune erosion of 30 to 100 meters, depending on the severity and the degree of instability of the beaches. This ultimately presents the shorefront property owner or other responsible individual/agency with three choices: (a) abandon the shoreline, (b) armor the shoreline in which case the beaches will gradually disappear, or (c) carry out fairly expensive beach nourishment programs.

As noted, the capability to provide the engineering and economic data to develop rational responses to the situations discussed above was clearly inadequate. It was difficult to partition erosion occurring to natural causes or human-related activities. Moreover, even if the characteristics of a storm were known precisely, only rudimentary approaches were available to predict the resulting erosion and the rate of recovery following the storm. The potential of this problem has been exacerbated by the predictions resulting from a recent comprehensive EPA study in which the rate of sea level rise over the next century is estimated to be between 10 to 30 times that occurring in the last century.

The strategy followed in the research project has been to develop a quantitative understanding of the mechanisms governing sediment transport processes and to formulate the understanding into numerical schemes that can be applied to realistic situations. The problem is complex and has resisted attempts of complete understanding. However, it is believed that substantial progress has been made and the basis has
been developed for numerous applications and also for future effective research programs.

The author of this report has been fortunate to have had the interest, insight, and motivation of the graduate students that contributed most significantly to this project. These included: David Kriebel, Brett Moore, Bill Dally, Osman Borekci, and Peter Williams.

II. BACKGROUND

A complete review of all substantial previous efforts that have been directed toward equilibrium profiles and beach profile evolution is beyond the scope of the present report; however, several closely related studies will be discussed briefly.

Bruun (1954) analyzed beach profiles along the Danish North Sea Coast and Mission Bay, California and found the following empirical relationship for the water depth, h, at an offshore distance, x,

\[ h(x) = Ax^{2/3} \]  \hspace{1cm} (1)

where A is a scale parameter. Bruun proposed two mechanisms responsible for the equilibrium beach profile. The first considered the onshore component of shear stress to be uniform and the onshore component of the gradient of transported wave energy to be constant. This resulted in an approximate equation of the form found empirically (Eq. (1)). The second mechanism was based on the consideration that the loss of wave energy is due only to bottom friction and that the loss per unit area is constant. A nonlinear wave theory was used with laboratory determined friction factors leading to the following

\[ h(x) = A' \frac{x^{2/3}}{T^{4/9}} \]  \hspace{1cm} (2)

where T is the wave period.

Bruun (1962) considered long-term erosion and proposed the following simple relationship expressing the beach recession, R, in terms of the increase in sea level, S, and the average beach slope, \( \tan \theta \), out to the location of limiting motion
This relation is based on the beach profile remaining the same in relation to the rising sea level.

Eagleson, et al. (1963) considered a balance between fluid and gravitational forces and developed a relationship for the equilibrium profile seaward of the surf zone. Comparison of the results with laboratory measurements were encouraging.

Edelman (1970) has developed geometric procedures for calculating the shoreline response due to storms. Basically, it is assumed that the response time of the beach is short relative to the storm time scale such that the beach profile relative to the instantaneous water level is the same as the initial (equilibrium) profile. Comparison of predictions with measured storm erosion (Chiu, 1981) shows that this method seriously overestimates the erosion (by factors of 4-10).

Hayden, et al. (1975) applied an empirical eigenfunction method of analysis to identify characteristic forms of 504 beach profiles along the Atlantic and Gulf of Mexico shorelines. This method has been extended and applied by Aubrey (1979) for California beaches. Moreover, Aubrey has correlated the various eigenfunctions with wave characteristics in an attempt to develop a predictive approach to beach profile response. The eigenfunction approach is purely empirical and does not address the processes associated with beach profile forms and mechanics of evolution.

Swart (1974) has conducted numerous laboratory studies of beach profiles and has analyzed these and other relevant data. The results for equilibrium beach profiles were presented by complicated empirical relationships. A method was also presented for beach profile evolution in which the offshore sediment transport, \( Q_s \), was presented as

\[
Q_s = K_* [(L_1 - L_2) \cdot t - W]
\]
in which $K_*$ is a rate constant, $W$ is the equilibrium spacing for two contours under consideration and $(L_2 - L_1)_t$ is the actual time-varying spacing of those two contours.

On the basis of a heuristic argument, Dean (1973) has proposed a fall velocity parameter, $F$, as significant in beach processes,

$$F \equiv \frac{H_b}{WT}$$

in which $H_b$ is the breaking wave height, $w$ is the fall velocity of the sediment and $T$ is the wave period. Consideration of bar formation further leads to the following two parameters

$$\text{Wave Steepness: } \frac{H_b}{L_o}$$

$$\text{Dimensionless Fall Velocity: } \frac{\pi w}{gT}$$

where $L_o$ is deep water wave length and $g$ is the gravitational constant. Comparison of 189 experiments showed that the parameters in Eq. (6) successfully identified conditions for which bars were formed for 89% of the data.

Hughes (1983) has proposed a scale relationship for physical models for beach and dune erosion. The relationship is based on equivalence of a fall velocity parameter in model and prototype and ratio of inertia to gravity forces. The modeling requirements allow for model distortion and include a geomorphological time scale. These relationships were evaluated against and compare favorably with dune erosion documented as a result of Hurricane Eloise in 1975.

Based on a series of small and large scale model tests, Vellinga (1982, 1983) has proposed a predictive procedure that has been shown to agree reasonably well with measured post-storm profiles associated with the 1953 and 1976 events in Holland. A reference profile is established for particular hydrographic and sediment characteristics. The results are then extended to "non-reference" conditions developed from the model studies. The method is completely empirical and strictly applicable only to a constant surge level over a five hour duration. Approximate
methods are presented accounting for storm durations in excess of five hours. Through modeling relationships, the effects of sediment size are taken into account.

van de Graaff (1983) has incorporated the methodology of Vellinga into a procedure for predicting the probability of dune erosion. The erosion is considered to be the result of seven independent parameters, each with a probability distribution of known characteristics. Based on the known probability characteristics of each of the seven variables contributing to the dune recession, two methods are presented for establishing the return period - dune recession relationship.

III. CHARACTERISTICS OF REALISTIC BEACH PROFILES

Beach profiles in nature are complex and dynamic, always changing due to altered conditions of tides, waves, winds, currents, or sediment supply. However, when considering many beach profiles, patterns emerge that are indicative of the general relationship to the different variables. Some of these characteristics and general response features are discussed below.

General Geometric Profile Characteristics

**Shape** - Beach profiles are generally characterized by a concave upward geometry. Thus the profile is steeper in the shallow water depths with the milder slopes occurring offshore. The beach face formed by the uprush and backwash of the waves is usually nearly planar.

**Form** - Beach profiles can be monotonic or may include one or more bars offshore. Usually storm waves will cause a bar to form which thereafter positions the larger breaking waves. Subsequent milder waves will cause the bar to move ashore in one or more sand "packets" termed "ridge and runnel systems".

**Scale** - The scale of beach profiles depends to a great extent on the sediment comprising the profile. Coarse sediment will form a steeper profile with a lesser tendency for bar formation than beaches composed of finer sediment.

**Sorting** - Waves are effective sorting agents tending to transport and deposit the coarser material in shallow water and depositing the
finer portions offshore. In some cases the beach profile includes the presence of a "step" feature at the base of the beach face. The coarsest material in the beach profile tends to collect at the base of the step which is a high energy environment due to the energy dissipation associated with the backwash. In cases where shell is present, the base of the step may be composed almost entirely of shell hash. Bars tend to contain the finer fraction of material available with the coarser fraction remaining as a lag product on the beaches. Winter bars form offshore of many California beaches leaving a cobble beach surface. During the milder summer months, the bars migrate ashore and a sand beach is formed again.

Effect of Water Level Changes

Many storms are accompanied by increases in mean water level due to storm surge and/or wave set-up. Additionally, some historically damaging storms have coincided with extreme astronomical tides. The effect of an increased water level is to cause the beach profile to be out of equilibrium and to increase the erosional potential of the storm waves. The increased water level affects the beach, on a short-term basis, in the same manner as sea level rise does on a long-term basis.

IV. EQUILIBRIUM BEACH PROFILES AND APPLICATIONS

A number of theories have been advanced attempting to describe the properties of and mechanisms associated with equilibrium beach profiles.

In the early phases of this study, a data set was located which consisted of more than 500 beach profiles ranging from the eastern tip of Long Island to the Texas-Mexico border, see Figure 1. Three fairly simple possible mechanisms were investigated relating the depth, h, to the distance offshore, x. Each of these three models predicted a profile of the following form

\[ h(x) = -Ax^m \]  

in which A and m are scale and shape parameters, respectively. Figure 2 presents normalized beach profiles for various m values. It is seen that for \( m < 1 \), the profile is concave upward as commonly found in nature. Figure 3 demonstrates the effect of the scale parameter, A.
Figure 1. Location map of the 502 profiles used in the analysis (from Hayden, et al., 1975).
DEFINITION SKETCH

Figure 2. Characteristics of dimensionless beach profile $\frac{h}{h_b} = (\frac{x}{w})^m$
for various m values (from Dean, 1977).
Figure 3. Equilibrium beach profiles for sand sizes of 0.2 mm and 0.6 mm
A(D = 0.2 mm) = 0.1 m$^{1/3}$, A(D = 0.6 mm) = 0.20 m$^{1/3}$. 
The three models and the associated values of $m$ are:

<table>
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<th>Model</th>
<th>$m$</th>
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<tr>
<td>1. Uniform Wave Energy Dissipation Per Unit Surface Area</td>
<td>0.40</td>
</tr>
<tr>
<td>2. Uniform Longshore Shear Stress</td>
<td>0.40</td>
</tr>
<tr>
<td>3. Uniform Wave Energy Dissipation Per Unit Water Volume</td>
<td>0.667</td>
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The derivations of each of the three models considered spilling breaking conditions, i.e. within the surf zone, the wave height, $H$, is proportional to the depth, $h$, through a proportionality constant, $\kappa$, i.e.

$$H = \kappa h$$  \hspace{1cm} (8)

where $\kappa$ is usually taken as 0.78.

The data from the 502 wave profiles were evaluated employing a least squares procedure to determine the $A$ and $m$ values for each of the profiles. The results of this analysis strongly supported a value of $m = 0.667$, (see Figure 4) i.e. the value associated with uniform wave energy dissipation per unit volume and as found earlier by Bruun (1954). The physical explanation associated with this mechanism is as follows. As the wave propagates through the surf zone, coherent wave energy is converted to turbulent energy by the breaking process. This turbulent energy is manifested as eddy motions of the water particles, thus affecting the stability of the bed material. Any model must acknowledge that a particular sand particle is acted on by constructive and destructive forces. The model here addresses directly only the destructive (destabilizing) forces. It was reasoned that the parameter $A$ depends primarily on sediment properties, and secondarily on wave characteristics, i.e.

$$A = F(\text{Sediment Properties, Wave Characteristics})$$  \hspace{1cm} (9)

where "$F()$" denotes "function of" and it would be desirable to combine wave and sediment characteristics to form a single dimensionless parameter.
Figure 4. Histogram of exponent $m$ in equation $h = Ax^m$ for 502 United States East Coast and Gulf of Mexico profiles (from Dean, 1977).
A portion of Mr. Brett Moore's M.S. Thesis (1982) was directed toward an improved definition of the scale parameter, A. Moore combined available laboratory and field data to obtain the results presented in Figure 5, thereby extending considerably the previous definition of A. Some of the individual beach profiles used in the development of Figure 5 are interesting. For example, Figure 6 presents the actual and best least squares fit to a beach consisting of "sand particles" 15-30 cm in diameter (approximately the size of a bowling ball). Figure 7 presents the same information for a beach reported to be composed almost entirely of whole and broken shells. Figure 8 shows a profile with a bar present resulting in one of the poorer fits to the data. It is emphasized that the analytical form (Eq. (7)) describes a monotonic profile.

V. CROSS-SHORE TRANSPORT MODELS

It has been noted that most equilibrium profiles correspond to uniform energy dissipation per unit volume with the scale of the profile represented by the parameter A which depends primarily on sediment characteristics and secondarily on wave characteristics, i.e.

\[ h(x) = Ax^{2/3} \]  

(10)

The parameter, A, and the uniform energy dissipation per unit volume, \( D^* \), are related for linear spilling waves by

\[ A = \left[ \frac{24}{5} \frac{D^*}{\rho g^{3/2} \kappa^2} \right]^{2/3} \]  

(11)

It can be shown that for the spilling breaker assumption and linear waves, the energy dissipation per unit volume, \( D \), is proportional to the product of the square root of the water depth and the gradient in depth,

\[ D = \frac{5}{16} \rho g^{3/2} \kappa^2 \frac{\partial h}{\partial x} \]  

(12)

Thus it is clear that an increase in water level such as due to a storm surge will cause wave energy dissipation to increase beyond the equilibrium value. It is also known that the beach responds by erosion
Figure 5. Beach profile factor, A, vs sediment diameter, D, in relationship $h = AD^{2/3}$ (modified from Moore, 1982).

From Hughes' Field Results

From Swart's Laboratory

Suggested Empirical Relationship

From Individual Field Profiles Where a Range of Sand Sizes Was Given
Figure 6. Profile P4 from Zenkovich (1967). A boulder coast in Eastern Kamchatka. Sand diameter: 150 mm - 300 mm. Least squares value of $A = 0.82 \text{ m}^{1/3}$ (from Moore, 1982).

Figure 7. Profile P10 from Zenkovich (1967). Near the end of a spit in Western Black Sea. Whole and broken shells. $A = 0.25 \text{ m}^{1/3}$ (from Moore, 1982).
Figure 8. Profile from Zenkovich (1967). Eastern Kamchatka. Mean sand diameter: 0.25 mm. Least squares value of $A = 0.07 \text{ m}^1/3$ (from Moore, 1982).
of sediment in shallow water and deposition of this sediment in deeper water (Figure 9). It therefore appears reasonable to propose as a hypothesis that the offshore sediment transport, $Q_s$, per unit width is given by

$$Q_s = K(D-D_*)$$

(13)

where $K$ is a rate constant that hopefully does not vary too greatly with scale. Moore (1982) evaluated this relationship using large scale wave tank data of Saville (1957) and found

$$K = 2.2 \times 10^{-6} \, m^4/N$$

(14)

Figure 10 presents comparisons of predicted cumulative erosion for various values of $K$ with the measured values obtained from Saville's wave tank tests.

VI. APPLICATION OF MODEL TO PREDICT BEACH PROFILE RESPONSE TO VARIOUS FORCING FUNCTIONS

In an effort to demonstrate model capabilities and to represent a broad range of beach profiles response, Moore (1982) explored the possibility of modeling various features of beach profiles, including longshore bars. This attempt required an improved description of the breaking wave process across the surf zone. For this purpose, a model developed by Dally (1980) was employed along with the sediment transport model given by Eq. (13) and the continuity equation

$$\frac{\partial h}{\partial t} = \frac{\partial Q_s}{\partial x}$$

(15)

Moore showed that the model successfully accounted for sea level rise effects (Figures 9 and 11) with the associated landward erosion and offshore deposition and that the model could account for the placement of a volume of sand on the profile, with the subsequent evolution to an equilibrium profile, see Figure 12.

Moore had limited success in modeling barred beach profiles. Depending on the type of initial breaking, a bar will develop with a
Figure 9. Model simulation of a 0.5 meter sea level rise and beach profile response with a relatively mild sloping beach (from Moore, 1982).
Figure 10. Effect of varying the sediment transport rate coefficient on cumulative erosion during the simulation of Saville's (1957) laboratory investigation of beach profile evolution for a 0.2 mm sand size (from Moore, 1982).
Figure 11. Model simulation of a 0.5 meter sea level rise and beach profile response with a steep 2 meter berm (from Moore, 1982).
Figure 12. Model simulation of beach nourishment for a 0.15 mm sand size (from Moore, 1982).
form similar to those developed in the laboratory or found in nature. One difficulty encountered was that with a pronounced bar present, the localized energy dissipation could be so severe as to cause instabilities. Moore applied a reasonable smoothing function to the energy dissipation and improved the stability of the computations. Figure 13 presents a comparison of a barred beach profile measured by Saville and that computed.

Moore also evaluated his model by comparison against measured profiles from the Nearshore Sediment Transport Study at Santa Barbara, California for the period January 21, 1980 to December 20, 1980. The initial and final prototype and predicted profiles are presented in Figure 14. The maximum and minimum (envelope) prototype and model profiles are presented in Figure 15. An empirical eigenfunction analysis was performed on the measured and predicted profiles. The first eigenfunction, the so-called "Mean Beach Function" is presented in Figure 16 where it is seen that reasonably good agreement occurs. The second or "Berm-Bar" eigenfunction is shown in Figure 17 where it is evident that the model results have the same general form, but are more irregular than the measured. The same general comments apply to the third eigenfunction, the "Terrace Function" presented in Figure 18.

VII. PREDICTION OF BEACH AND DUNE EROSION DUE TO SEVERE STORMS

Mr. David Kriebel conducted the last component of work on the project to be reported here as a Master's thesis. Most of the previous work was incorporated and considerable original contributions were developed into a two-dimensional predictive model of beach and dune erosion for single storm events and for long-term scenarios in which many storms occur.

Profile Schematization

The profile was schematized as a series of depth contours, \( h_n \), the locations of which are specified by coordinates, \( x_n \), measured from an arbitrary baseline, see Figure 19. The profile is thus inherently monotonic and at each time step, the \( x_n \) values of each of the active contours is updated.
Figure 13. Comparison of the beach profile from the model and Saville's laboratory 0.2 mm sand after 25 hours (from Moore, 1982).
Figure E4. Initial (January 21, 1980) and final (December 20, 1980) measured and predicted beach profiles. Leadbetter Beach, Santa Barbara, California (from Moore, 1982).
Figure 15. Model versus prototype beach profile envelopes (i.e., maximum and minimum water depths at each location along the beach profile) for the entire year (1980) (from Moore, 1982).
Figure 16. Comparison of Eigenfunction analysis results performed on the predicted and measured beach profiles: First Eigenfunction, mean-beach function (from Moore, 1982).
Figure 17. Comparison of Eigenfunction analysis results performed on the predicted and measured beach profiles: Second Eigenfunction, bar-berm function (from Moore, 1982).
Figure 18. Comparison of Eigenfunction analysis results performed on the predicted and measured beach profiles: Third Eigenfunction, terrace function (from Moore, 1982).
Figure 19. Model representation of beach profile, showing depth and transport relation to grid definitions (from Kriebel, 1982).
Governing Equations

As in most transport problems, there are two governing equations. One is an equation describing the transport in terms of a gradient or some other feature. The second is a continuity or conservation equation which accounts for the net fluxes into a cell.

As discussed previously, the offshore transport is defined by Eq. (13) in terms of the excess energy dissipation per unit volume. Specifically, in finite difference form

\[ \mathcal{D}_{n+1} = k_D \frac{h_{n+1}^{5/2} - h_n^{5/2}}{(h_{n+1}^{1/2} + h_n^{1/2})(x_{n+1} - x_n)} \]  

(16)

where

\[ k_D = \frac{\gamma}{4} \frac{k^2 \sqrt{g}}{ \phi} \]  

(17)

The conservation equation is

\[ \Delta x_n = \frac{K \Delta t}{\Delta h} \left( \mathcal{D}_n - \mathcal{D}_{n+1} \right) \]  

(18)

Method of Solution of Finite Difference Equations

A number of methods could be employed for solving Eqs. (13) and (18). For example, explicit methods would be fairly direct and simple to program; however, the maximum time increment would be relatively small resulting in a program which is quite expensive to run. Implicit methods are somewhat more difficult to program, but have the desirable feature of remaining stable with a much greater time step. Because of the planned application to long-term simulation in which for a 500 year time period over three hundred storms would be modeled, each with an erosional phase of six to twelve hours, an implicit method was adopted. This method will not be described in detail here except to note that a double sweep approach is used in which the \( Q_{n} \) values and the \( x_n \) values are updated simultaneously at each time step. For \( \Delta h \) values of 1 ft, and a time step of thirty minutes, the system of equations was stable.
The boundary conditions used were somewhat intuitive. At the shoreward end of the system, erosion proceeded with a specified slope above a particular depth, $h_\ast$. The depth, $h_\ast$, is the depth that the equilibrium slope and the slope corresponding to the beach face are the same. Thus a unit of recession of the uppermost active contour causes an erosion of the profile above the active contour that is "swept" by this specified slope. This material is then placed as a source into the uppermost active contour. The offshore boundary condition is that the active contours are those within which wave breaking occurs. If an active contour extends seaward, thereby encroaching over the contour below to an extent that the angle of repose is reached, the lower contour (and additional lower contours if necessary) are displaced seaward to limit the slope to that of the angle of repose.

Application of Method to Computation of Idealized Beach Response

Kriebel (1982) carried out computations for a number of idealized cases, some of which are reviewed below.

Response to Static Increased Water Level - Figure 20 presents the beach recession due to a static increase in water. The beach responds as expected. In the early response stages, the rate of adjustment is fairly rapid with the latter adjustments approaching the equilibrium recession in an asymptotic manner. Of special relevance is that the response time to equilibrium is long compared to the duration of most severe storm systems, such as hurricanes. The form of the response presented in Figure 20 is reminiscent of that for a first order process in which the time rate of change of beach recession, $R$, is represented as

$$\frac{dR}{dt} = -\alpha R \quad (19)$$

for which the solution is

$$\frac{R(t)}{R_\infty} = (1-e^{-\alpha t}) \quad (20)$$

Figure 21 presents a comparison of the response from the numerical model and Eq. (20). This similarity forms the basis for a very simple and approximate numerical model of beach and dune profile response. Such a model has been developed but will not be presented here.
Figure 20. Characteristic form of berm recession versus time for increased static water level (from Kriebel, 1982).
Figure 21. Comparison of asymptotic berm recession from model (---) and as calculated by Eq. (20) (○ ●).
Effects of Various Wave Heights - Considering a common increased water level, but storms with different wave heights, the larger wave heights will break farther offshore causing profile adjustments over a greater distance and thus a greater shoreline recession. Simulations were carried out to examine evolution of the beach under different wave heights with the results presented in Figure 22. As expected the greater shoreline recessions are associated with the larger wave heights. Surprisingly, however during the early phases of the evolution, the larger wave heights do not cause proportionally larger erosions. Thus, for storms of short duration, the sensitivity of the maximum erosion to breaking wave height may not be large.

Effects of Various Storm Tide Levels - The counterpart to the previous case is that of a fixed wave height and various storm water levels. The results of these simulations are presented in Figure 23. In contrast to the previous case, the various storm tide levels cause recession rates in the early stages of the process which are nearly proportional to the storm water level.

Effect of Sediment Size on Berm Recession - The effect of two different sediment sizes on amount and rate of berm recession is shown in Figure 24. The equilibrium recession of a coarser material is much less; however, the equilibrium is achieved in a much shorter time than that for the finer sediment. The explanation for the lesser equilibrium erosion for the coarser material is that since the beach is steeper, the waves break closer to shore and thus less material is required to be transferred offshore to establish an equilibrium profile out to the breaking depth (considered to be the limit of motion). Presumably the explanation for the slower approach to equilibrium for the finer material is that, as will be shown by consideration of the initial and equilibrium profile geometries, a much greater volume of sediment must be moved a greater distance to establish equilibrium.

Effect of Storm Duration - The effect of storm duration on shoreline recession was investigated by considering a fixed wave height and an idealized storm tide variation, expressed as
Figure 22. Effect of breaking wave height on berm recession (from Kriebel, 1982).
Figure 23. Effect of static storm surge level on berm recession (from Kriebel, 1982).
Figure 24. Effect of sediment size on berm recession. (from Kriebel, 1982)
\[ \eta = 1.2 \cos^2 \left( \frac{\sigma(t-18)}{2} \right), \quad |t-18| \lesssim \frac{T}{2} \]

\[ = 0, \quad |t-18| > \frac{T}{2} \quad (21) \]

in which \( T \) (\( \equiv 2\pi/\sigma \)) is the total storm duration in hours. The results are presented for three storm durations in Figure 25. For the shortest storm duration (\( T = 12 \) hours), the potential volume eroded is approximately 70 \( m^3/m \) whereas the computed actual maximum volume eroded is 10 \( m^3/m \). With increasing storm tide duration, the computed actual maximum volume eroded increases. Tripling the storm tide duration to 36 hours doubles the maximum volume eroded to 20 \( m^3/m \). It is noted that this is only approximately 28% of the potential volume eroded, again underscoring the likelihood that most storms will only reach a fraction of their potential erosion limit. This feature also highlights the significance of cumulative effects of sequential storms and of the need to better understand the recovery process (especially the rates), a portion of the cycle not addressed in this project.

**Application of Method to Long-Term Beach and Dune Response Simulations**

The previous section has described the application of the model to idealized examples of beach and dune response. The model can also be applied to more realistic situations in which the initial beach and dune conditions are specified along with time-varying waves and tides.

**Evaluation of Method by Hurricane Eloise Erosion Data** - Kriebel carried out an evaluation of the method by comparing erosion computations for Hurricane Eloise (1975) with measurements reported by Chiu (1977). Although the wave and tide conditions were not measured along the beaches of Bay and Walton Counties (Florida) of interest, some tide data were available and wave heights were estimated. Erosion was computed for twenty combinations of dune slope, wave height and peak surge. It was found that the volumetric erosion ranged from 21 to 38 \( m^3/m \) compared to average measured values of 18 to 20 \( m^3/m \) for Bay and Walton Counties, respectively and an average of 25 \( m^3/m \) near the area of peak surge. Although the predicted values are somewhat larger than the observed, Chiu (1977) states that the beaches had started to recover at the time of the post-storm surveys, with approximately 5 \( m^3/m \) of sand.
Figure 25. Comparison of the effects of 12, 24, and 36 hrs. storm surge on volumetric erosion (from Kriebel, 1982).
having returned to the beach. Thus the maximum eroded volume would be 30 m$^3$/m compared to a maximum calculated value of 38 m$^3$/m, a difference of approximately 27%. This reasonably close agreement was considered adequate recognizing the uncertainty in the storm tide employed in the computations; therefore no further calibration of the model was considered warranted. It is of interest that the erosion potential associated with the peak tide is approximately nine times that predicted for the time-varying conditions included in the computations. This again reinforces the fact that most storms in nature cause only a fraction of the potential erosion associated with the maximum conditions in the storm.

**Long-Term Simulation** - With the model reasonably verified for the Bay and Walton Counties area of Florida, a long-term simulation of beach and dune erosion was carried out. The hurricane wind and pressure fields were idealized in accordance with a representation published by Wilson (1956). The five idealized hurricane parameters

\[
\begin{align*}
\Delta p &= \text{Maximum Pressured Deficit} \\
R_{\text{max}} &= \text{Radius to Maximum Winds} \\
V_F &= \text{Hurricane System Translational Speed} \\
\beta &= \text{Hurricane Translational Direction} \\
y_F &= \text{Landfall Point}
\end{align*}
\]

were selected by a Monte Carlo method in accordance with the historical characteristics of hurricanes in the general area. For each hurricane, the storm tide was calculated using the Bathystrophic Storm Tide Model of Freeman, Baer and Jung (1957). With the time-varying storm tide and wave height calculated, the beach and dune model was applied until maximum erosion was achieved. As the recovery mechanism is not yet understood to a degree for realistic modelling and because hurricanes occur approximately on a biennial basis, the erosion for successive hurricanes was assumed to commence from a fully recovered condition. This is clearly an approximation as the recovery process occurs at several rates of magnitude slower than the erosion process. Study of some recovery stages from severe storms has shown that up to seven years may be required to achieve approximately 90% recovery. The duration required for recovery from milder storms would, of course, be less.
Figure 26 presents a "flow chart" describing the elements of the long-term simulation. In the Bay-Walton Counties area, hurricanes making landfall within ± 150 n.mi. of these counties were considered requiring a total of 393 hurricanes to simulate a 500 year record. The return periods associated with various dune recessions as determined from the simulations are presented in Figure 27. As examples, the dune recessions for return periods of 10, 100 and 500 years are 4 m, 12 m and 18 m, respectively. Based on these results, Hurricane Eloise is judged to represent a 20 to 50 year erosional event; however based on results from a storm surge analysis, Hurricane Eloise was a 75 to 100 year coastal flooding event.

It is also possible to present the results of the erosion simulations in a manner that is of maximum relevance to individuals or agencies responsible for shoreline management. This type of presentation is demonstrated for the Bay-Walton County area in Figure 28. This plot includes the contributions from storms and sea level rise. As examples, without any erosion mitigation measures within the next 50 years, the erosion due to sea level rise (regarded as a certainty or probability of 100%) is expected to be approximately 15 ft. Within 50 years, the probability of erosion occurring to a distance of 40 ft is 85% and for distances of 60 and 80 ft, the corresponding probabilities are 32% and 9%, respectively. Through the use of figures such as these it would be possible to weigh the costs of certain erosion control measures against the potential of damage if those measures are not carried out.

These procedures provide, for the first time, a basis for conducting the necessary technical studies to implement the erosion component calculations of the Flood Insurance Act of 1973 which provides for the application of methodology to provide the basis for insurance rates for flooding and erosion coastal hazards. Although the flooding component of this act has been implemented, the erosion component has not.

It is noted that the State of Florida Division of Beaches and Shores of the Department of Natural Resources presently utilizes the erosion simulation model of Kriebel and simplifications thereof in the
Figure 26. Flow diagram of N-year simulation of hurricane storm surge and resulting beach erosion (from Kriebel, 1982).
RETURN PERIOD IN YEARS

PROBABILITY OF OCCURRENCE OR EXCEEDANCE

Figure 27. Average frequency curve for dune recession, developed by Monte Carlo simulation, Bay-Walton Counties, Florida (from Kriebel, 1982).
Figure 28. Probability or risk of dune recession of given magnitude occurring at least once in N-years, Bay-Walton Co., Florida (from Kriebel, 1982).
establishment of the Coastal Construction Control Line and in the consideration of various applications for coastal construction permits.

VIII. OTHER APPLICATIONS AND PUBLICATIONS

In addition to the above comprehensive contributions by Moore (1982) and Kriebel (1982), a number of publications and applications have resulted from this project.

The design of beach nourishment projects has been discussed by Maurmeyer and Dean (1980) with specific reference to the placement of sand to minimize overtopping by waves. In addition, an examination was carried out of the effect of sand size on the usable width of beach after reconfiguring of the profile by waves of different heights. Methods were presented for calculating the wave overtopping as a function of volumes and types (sizes) of beach sand placed. Figure 29 presents, for various sediment characteristics, the required nourishment volumes to advance the shoreline seaward a distance of 300 ft.

The effect of wave steepness and fall velocity parameter on volume of material stored in the offshore bar was examined by Dean (1982). A series of systematic wave tank experiments by Coxe (1978) was employed to develop a dimensionless relationship for the bar volume. Figure 30 presents the bar volume as a function of the square of the excess wave height above that required for incipient bar formation.

A number of laboratory and field experiments had been carried out by various investigators to quantify the immersed sediment transport rate, \( I_L \), in terms of the so-called longshore energy flux at breaking, \( P_L \), i.e.

\[
I_L = K' P_L
\]  

(22)

It was found that the laboratory derived values of \( K' \) were significantly lower than the field values and this was taken as grounds that serious scale effects were present in the modeling of longshore sediment transport. Examination of the model versus field conditions demonstrated a scale ratio of approximately 1:10 for the waves, but a scale ratio of approximately 1:3 to 1:1 for the model sediment. Thus
Figure 29. Nourishment volumes required versus effective wave height for various native and filled sediment characteristics and considerations, shoreline advancement = 300 ft. (from Maurmeyer and Dean, 1980).
Figure 30. Relationship between volume of sand stored in bar versus wave height above that required for incipient bar formation. (from Dean, 1983)
the model sediment scaled to the prototype is from three to ten times larger than the field sediment. Dean (1983) recommended that the scaling be conducted in accordance with the following parameter which incorporates both sediment and wave characteristics.

\[
\frac{H_b}{g \frac{w}{2}}
\]  

(23)

where \(H_b\) is the breaking wave height and \(w\) is the fall velocity of the sediment. Without presenting the details, Figure 31 demonstrates that the use of the parameter \(gH_b/w^2\) allows unification of the laboratory and field results.

The principles of beach nourishment were reviewed by Dean (1983). Included were the effects of sediment size on total volumes required and computational procedures to determine reduction in sand volumes required through application of a "perched beach" concept, see Figure 32. Also described were the merits of stabilization of beach nourishment projects by structures.

Dean and Maurmeyer (1983) presented models for long-term response to sea level rise. Models were presented in graphical form for a natural beach profile and a beach profile limited by a seawall. Figures 33 and 34 present the results for natural and seawalled beaches, respectively. It is seen from Figure 33 that, for a fixed breaking wave height, the berm recession for the natural beach increases with increasing storm tide. However, for the seawalled case, with increasing storm tide and fixed wave height, the deepening at the base of the seawall increases, reaching maximum, then with further increases in storm tide, decreases. The interpretation is that initially an increase in storm tide requires substantial erosion to meet the demand of maintaining the offshore profile the same relative to the fixed water level. However, increasing storm tides will cause the horizontal extent of the region requiring deposition to decrease to zero, thereby resulting in zero scour. At the limit, where no wave breaking occurs, (all the wave energy is reflected), this approximate method predicts that no erosion would occur. Of course, this result is not completely realistic. A model for barrier island response was presented which
Figure 31. Suggested variation of \( K \) with \( gH/w^2 \), prototype and laboratory data (from Dean, 1983).
Figure 32. Perched beach, demonstrating nourishment volumes saved (from Dean, 1983).
Figure 33. Isolines of dimensionless berm recession, $R'$, vs dimensionless storm breaking depth, $h'_b$, and dimensionless storm tide, $S'$, $m = 2/3$. (from Dean and Maurmeyer, 1983)

Figure 34. Isolines of dimensionless seawall toe scour, $h'_w$ vs dimensionless storm tide, $S'$ and dimensionless breaking depth $h'_b$. (from Dean and Maurmeyer, 1983)
accounts for the offshore transport of sand on the ocean and bay sides and also the upward growth of the barrier island to maintain elevation relative to the rising sea level, Figure 35.

The equilibrium beach profile represented by Eq. (10) is extremely simple and has proven useful in a number of applications. It is recalled that the basis for this profile is that the sediment particle can withstand a certain level of destructive forces (energy dissipation resulting in turbulent fluctuations); if the destructive forces exceed this level, the profile will be remolded by reducing the local slope and/or depth to again achieve equilibrium. One unrealistic feature of this consideration is that gravity is not recognized as a destructive force. Thus it is implicitly assumed that the slopes are so mild that the gravitational forces are small compared to those induced by turbulence. Inspection of Eq. (10) shows that the slope of the profile is infinite at the mean water line. To account for the effect of non-mild slopes, the equilibrium energy dissipation per unit volume is modified to include the effect of gravitational forces

$$p_* = p_*^o \left[ 1 + \frac{\partial h}{\partial x} \right]$$

where $p_*^o$ is the equilibrium energy dissipation on a flat slope and $\left( \frac{\partial h}{\partial x} \right)_o$ is the limiting slope for the sand. Equating the above to the wave energy dissipation per unit volume and simplifying, the solution relating water depth, h, to distance from shore, x, is

$$\frac{5}{24} \rho g \frac{3}{2} \kappa \frac{2}{h} \frac{3}{2} + \frac{p_*^o h}{\left( \frac{\partial h}{\partial x} \right)_o} = p_* x$$

Note that the first term on the left hand side and the term on the right hand side represent the solution developed earlier. Also, the second term on the left hand side dominates in very shallow water and predicts a uniform slope which is in accordance with beach face descriptions.

Considering a nearshore slope of 1:15 (a reasonable value for a sand size of 0.2 mm), Figure 36 compares the beach profile with and without inclusion of the gravitational term.
Figure 35. Generalized shoreline response model due to sea level rise. Applicable for a barrier island system which maintains its form relative to the adjacent ocean and lagoon water levels (Dean and Maurmeyer, 1983)
Figure 36. Comparison of equilibrium beach profile with and without gravitational effects included. $A = 0.1 \text{ m}^{1/3}$ corresponding to a sand size of 0.2mm.
IX. SUMMARY AND CONCLUSIONS

The characteristics of equilibrium beach profiles in nature have been found to be of the approximate form

\[ h(x) = Ax^{2/3} \]  

(26)

where A is a scale parameter depending primarily on sediment size, and secondarily on wave characteristics. Eq. (26) was shown to be consistent with a spilling breaking model and a uniform wave energy dissipation per unit water volume.

Eq. (26) has been applied to the quasi-static case of predicting shoreline recession due to sea level rise along natural shorelines and deepening in front of seawalled profiles. Additionally, the profile has been applied to design problems in beach nourishment projects, including beach widths associated with volumes and diameters of sand used. The particular case of a perched beach has been treated.

The equilibrium beach profile results have been extended to the case of non-equilibrium by proposing the following offshore sediment transport relationship

\[ Q_s = K(D - D^*_e) \]  

(27)

in which \( D \) is the wave energy dissipation per unit volume and, \( D^*_e \) represents the equilibrium value. Eq. (28) has been combined with the continuity equation to represent a number of problems of interest, including: many idealized examples, bar formation (which requires a more realistic breaking model than the spilling breaker model), the forms of beach change and dune erosion by severe storms, including simulation of a 500 year period for one location. All of the results appear realistic and encouraging.

Several problems on which future research should be focused include:

(1) The recovery phase following erosion which is known to proceed at a much slower rate,
(2) Comparison of predicted and measured shoreline response, including normal seasonal storms and severe events,
(3) The effect of wave characteristics on the parameter, A,
(4) The development of improved wave breaking models,
(5) The mechanisms and causes of bar formation, and
(6) The effects of natural offshore rock structures on shoreline response.

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APPENDIX I REFERENCES


