A Simple Spreadsheet Model to Incorporate Seasonal Growth into Length-Based Stock Assessment Methods

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Abstract

The paper describes a method by which seasonal growth can be incorporated into length-converted catch curves and cohort analyses using a spreadsheet. The method is based on calculating the length of fish using seasonal growth parameters on a daily basis. A LOOKUP function is then used to find the age corresponding to the length.

Introduction

Over the last decade, fisheries biologists working in tropical waters became increasingly familiar with “length-based fish stock assessment”, thanks to the development of easy-to-use software (such as LFSA, ELEFAN, FISAT, and LFDA); good training manuals (Sparre and Venema 1992, 1998); the worldwide FAO/DANIDA training course in Tropical Fish Stock Assessment; and the availability of relatively cheap computers.

Most of the traditional stock assessment methods work with age composition data, whereby annuli (on otoliths, scales and other bones) are used to estimate growth. However, in tropical waters this type of age reading is almost impossible and stock assessment became much easier with the development of length-based methods.

In principle, length-based fish stock assessment is based on the conversion of length into age, where it is assumed that the fish is growing according to the von Bertalanffy Growth Function (VBGF). It is generally accepted that in temperate waters the growth of fish has strong seasonal fluctuations, mainly due to fluctuations of temperature and/or food supply (Shul’mann 1974). Strong seasonal fluctuations in growth also exist in tropical waters (Daget and Ecouin 1976; de Graaf and Ofori-Danson 1997; de Graaf 2003). The use of a seasonal version of the VBGF has been discussed extensively by several authors (Pauly and Ingles 1981; Pauly et al. 1992; Longhurst and Pauly 1987; Sparre 1990) and seasonal versions of standard analytical models, such as the yield-per-recruit method (Sparre 1991) and length-converted catch curves (Pauly 1990), were developed and incorporated in the software packages. However, these models are very basic methods of stock assessment and provide only a first approximation of the status of fish stocks and the impact of fishing mortality. More detailed information is required for the formulation of fisheries management strategies. This is available through Virtual Population Analysis (VPA) or length-based cohort analysis and Thompson and Bell models (Sparre and Venema 1992). Pauly et al. (1987) used the seasonal version of the VBGF to slice the cohorts in a time-based VPA for Peruvian Anchoveta. This technique has been incorporated into the DOS version of FISAT, but the length-based VPA in FISAT is still non-seasonal. Therefore, the analysis of data with a spreadsheet is dependent on non-seasonal models that will give distorted results if the growth is seasonal.

This paper presents an approach in which seasonal growth can be incorporated in some major fish stock assessment methods using a spreadsheet.

Non Seasonal and Seasonal Growth

The basic tool for length-based methods is the conversion of length-based data into age-based data. Traditionally the VBGF is used for the conversion of length into age in length-based fish stock assessments. The non-seasonal version of VBGF takes the form:

\[ L_t = L_\infty \left(1 - e^{-K(t-t_0)}\right) \]

(1)

Where

\[ L_t \] Length at time \( t \)
\[ L_\infty \] L infinitive or asymptotic length
\[ K \] growth parameter
\[ t_0 \] T zero, or time when the fish are born or entered in the system
The growth rate at any point in the lifespan of the fish can be calculated as:

\[
\frac{dL}{dt} = K(L_\infty - L_t)
\]

(2)

Conversion of length into age is done with the inverse VBGF:

\[
t(L_t) = t_o - \frac{1}{K} \ln \left(1 - \frac{L_t}{L_\infty}\right)
\]

(3)

In conclusion, the conversion of length-based data into age-based data for growth without seasonality is rather straightforward. However, this is not the case where there are seasonal fluctuations in growth. The seasonal version of the VBGF (Somers 1988) has the following form:

\[
L(t) = L_\infty \left[1 - e^{-K(t-t_o)}\sin^2 \pi (t-t_s)\right]
\]

(4)

Where:
- \(L_t\) Length at time \(t\)
- \(L_\infty\) L infinitive
- \(K\) growth rate parameter
- \(t_o\) T-zero
- \(t_s\) the onset of the first oscillation relative to \(t = 0\), or \(t_s = \text{Winter point} + 0.5\)
- \(C\) the intensity of the (sinusoid) growth oscillations

The parameter \(C\) is important as it determines the intensity of the seasonal growth. When \(C = 0\), seasonal growth is absent and the equation equals the standard VBGF. At \(C = 1\), growth is highly seasonal and comes to a standstill once a year at the winter-point. Intermediate values of \(C\) indicate growth reduction during the winter, but growth never completely stops.

The differences in growth and growth rate (\(dL/dt\)) for both versions of the VBGF are illustrated in Figure 1.

The growth rate (\(dL/dt\)) is the differential of the VBGF and is solved with mathematical software (Heck 2003) into the form:

\[
\frac{dL}{dt} = -L_\infty \left(K - CK \cos \left(2\pi(t - t_s)\right)\right) e^{-K(t-t_o)}
\]

\[
\frac{CK(\sin(2\pi(t-t_s)) - \sin(2\pi(t_0-t_s)))}{2\pi}
\]

(5)

However, an inverse seasonal VBGF does not have a direct solution due to the fact that on the right side of the VBGF \(t\) is found two times in the exponential factor and can only be solved "numerically".

### Estimating age when length is known in a seasonal VBGF

Converting length into age for the seasonal VBGF using a numerical mathematical approach can be done using the Solver function in Excel (Lleonart, pers. com). However, as the Solver function cannot be used in a multiple form, this does not lead to a simple and practical solution. Therefore, we looked at how we could approach the value of \(t\) for a given value of \(L\) as follows:

- Calculate the length of the fish with daily intervals for a set of given parameters of the seasonal VBGF.
- Search in this data set the length to be converted into age.
- Take in this data set the corresponding age value or \(t\).

In a spreadsheet, this can be done with a LOOKUP function, as explained below.

### Step 1. calculating length

Figure 2 presents the setup of a spreadsheet used to calculate length with seasonal growth. To facilitate building, the VBGF has been separated first into smaller blocks \([K(t-t_s); CK/2\pi; \sin2\pi(t-t_s); \sin2\pi(t_0-t_s)]\) that
are combined again in the column cells I10:I16 to calculate $L(t)$. In this example from Bangladesh, the fish are born on May 15$^{th}$ ($t_0=0.37$) and have the highest growth in June and a reduced growth in December/January. In the column cells D10:D16 the age of the fish is calculated and together with the column cells I10:I16 they are the basic input for the conversion of length into age. Figure 3 presents the formulae for the different spreadsheet cells. Only the first few days’ growth is presented in the example. With the Excel cell copying feature, the formulae in each relevant column can be copied down to row 1104 in order to cover a period of three years.

Step 2. inserting the lookup function

With the LOOKUP function in Excel, the instruction is given to search for an indicated value in a column or row and, once the value is found, to provide a corresponding value from another column or row. In our case, LOOKUP searches for a given length in the column where the calculated length data are stored and returns the corresponding age for this length from the column where the ages are stored. The LOOKUP function has three entries:

1. **Lookup_value** is a value that LOOKUP searches for in Lookup_vector and can be a number, text, or a logical value. In our case it is the $L(t)$.

2. **Lookup_vector** is a range that contains only one row or one column of numbers, text or logical values, placed in ascending order. In our case it is the column with the ‘length data’.

3. **Result_vector** is a range that contains only one row or column of numbers, text or logical values and has the same size as the Lookup_vector. In our case it is the column with the ‘age data’.

An example of conversion of length into age with the LOOKUP function is presented in Figure 4. In cells M10: M22 and N10:N22 the length classes are entered with a 1 cm interval. The mid-length of each length class is calculated in the cells O10:O22. With the LOOKUP function entered in cells P10:P22, the age of each mid-length is estimated in the spreadsheet. In cell O10, the mid-length of the interval 0-1 cm is calculated (0.5 cm), and with the LOOKUP function the column $L(t)$ is searched for the first value approaching 0.5. Once found, in cell L16, it takes the corresponding age value from column K and enters

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1 In the formula in the spreadsheet $t_s$, or the onset of the first oscillation relative to $t = 0$ is used. Users of FISAT are more familiar with the Winterpoint WP = $t_s + 0.5$
this value in cell P10. In our example the first value, approaching 0.5 cm, is 0.469 cm on day 6, with an age of 0.016 years. It could be argued that the method is rather inaccurate. However, in the example, the fish are growing with steps of one day. Reducing this to steps of 0.25 day improves the accuracy considerably but also increases the size of the spreadsheet.

The formulae for the different cells are presented in Figure 5.

Length-based methods are often used when length composition data for the total fishery is available for a one year period only. This is often the case in fish stock assessment programs in developing countries where the funds for continuous monitoring of the fishery are lacking. Catch curves and cohort analysis can be applied under these conditions as their basic assumption is the constant parameter system. The picture presented by all length classes caught during one year reflects that of a single cohort during its entire life span (Sparre and Venema 1992).

**Linearized Length-converted Catch Curve**

The construction of a catch curve is the most common approach to estimate the total mortality of a cohort. Details of the method are well presented by Sparre and Venema (1992) and Gayanilo and Pauly (1997) and are only briefly summarized here. Assuming constant recruitment and constant mortality the length converted catch curves takes the form:

\[
\ln \left( \frac{C_i}{dt_i} \right) = a + Zt_i^*.
\]

Where

- \( C_i \) catch number of length class i
- \( dt_i \) time needed for the fish to grow through length class i
- \( Z \) total mortality
- \( t_i^* \) age of the mid-length of length class i
- \( a \) constant

For non-seasonal growth \( dt_i \) is estimated from:

\[
dt_i = \left( \frac{1}{K} \right) \ln \left[ \frac{L_i - L_{i-1}}{L_i - L_{i-1}} \right]
\]

and \( t_i^* \) is estimated from:

\[
t_i^* = \left( \frac{L_0}{L_i} \right) \ln \left[ 1 - \left( \frac{L_i}{L_0} \right)^{-\frac{1}{Z}} \right]
\]

Sparre (1990) and Pauly (1990) clearly demonstrated that the total mortality is overestimated by the traditional catch curve if seasonal growth is not accounted for. The reason is that \( dt_i \) and \( t_i^* \) depend not only on length but also on the time of the year if growth is seasonal. Pauly (1990) developed a method using the parameters of the seasonal VBGF to identify a number of pseudo cohorts to resolve this problem. This method is incorporated into FISAT. As it is rather complicated to apply this method in a spreadsheet, we explain how a catch curve can be made with the LOOKUP function to provide results comparable to those with Pauly’s method. We illustrate this with an example of *Puntius sophore* from Bangladesh.

The number of *Puntius sophore* caught in a one-year stock assessment program for the different length classes is presented in Table 1.

**Table 1. Length frequency distribution and estimated growth parameters of *Puntius sophore* as obtained through a one-year stock assessment program.**

<table>
<thead>
<tr>
<th>L1 (cm)</th>
<th>L2 (cm)</th>
<th>Mid length (cm)</th>
<th>Catch numbers</th>
<th>Growth parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0</td>
<td>1.0</td>
<td>0.5</td>
<td>10</td>
<td>Linf 13</td>
</tr>
<tr>
<td>1.0</td>
<td>2.0</td>
<td>1.5</td>
<td>250</td>
<td>K 1.3</td>
</tr>
<tr>
<td>2.0</td>
<td>3.0</td>
<td>2.5</td>
<td>590</td>
<td>to 0.37</td>
</tr>
<tr>
<td>3.0</td>
<td>4.0</td>
<td>3.5</td>
<td>70</td>
<td>ts 0.5</td>
</tr>
<tr>
<td>4.0</td>
<td>5.0</td>
<td>4.5</td>
<td>520</td>
<td>C 1</td>
</tr>
<tr>
<td>5.0</td>
<td>6.0</td>
<td>5.5</td>
<td>2160</td>
<td></td>
</tr>
<tr>
<td>6.0</td>
<td>7.0</td>
<td>6.5</td>
<td>3830</td>
<td></td>
</tr>
<tr>
<td>7.0</td>
<td>8.0</td>
<td>7.5</td>
<td>1970</td>
<td></td>
</tr>
<tr>
<td>8.0</td>
<td>9.0</td>
<td>8.5</td>
<td>1150</td>
<td></td>
</tr>
<tr>
<td>9.0</td>
<td>10.0</td>
<td>9.5</td>
<td>490</td>
<td></td>
</tr>
<tr>
<td>10.0</td>
<td>11.0</td>
<td>10.5</td>
<td>270</td>
<td></td>
</tr>
<tr>
<td>11.0</td>
<td>12.0</td>
<td>11.5</td>
<td>50</td>
<td></td>
</tr>
<tr>
<td>12.0</td>
<td>13.0</td>
<td>12.5</td>
<td>30</td>
<td></td>
</tr>
</tbody>
</table>

**Figure 5.** The cell formulae for the conversion of length into age with a LOOKUP function.
The distribution is believed to be representative for the overall population structure as the fish were caught with non-selective gears.

We have to calculate $dt$, the time needed for the fish to grow through length class $l$ in order to construct the catch curve. For each length class this is done with the LOOKUP function as follows:

$$dt = \text{age } L2 - \text{age } L1 \quad (8)$$

Conversion of the mid-length into age has been demonstrated in the previous paragraph and construction of the catch curve then becomes straightforward (Figures 6 and 7).

In column $S$, delta $t$ ($dt$) is calculated and $\ln(C/dt)$ is calculated in column $T$. Plotting $\ln(C/dt)$ against the age of the mid-length (column $P$) gives the catch curve (Figure 8) indicating a total mortality of 4.57 year$^{-1}$, which is similar to the results of analysing the data in FISAT with the method of Pauly (1990).

Cohort or Virtual Population Analysis

Cohort or Virtual Population Analysis use the number of fish caught during commercial fishing operations to estimate historic fishing mortality.

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2 Adapted from Gayanilo and Pauly (1997) and King (1995).
and stock numbers in a cohort of fish and is again based on the constant parameter system.

The number of fish surviving from one year \( N_t \) to the next year \( N_{t+1} \) is given by:

\[
N_{t+1} = N_t \times e^{-[F_t \times M]} \tag{9}
\]

The number of dying fish is therefore:

\[
N_t \times \left(1 - e^{-Z} \right) \tag{10}
\]

The catch \( C_t \) is the proportion dying owing to fishing, and may be estimated from the catch or the Baranov (1926) equation:

\[
C_t = \left[\frac{F_t}{Z}\right] \times N_t \times \left(1 - e^{-[F_t \times M]} \right) \tag{11}
\]

Combining the different equations will give the Gulland (1965) equation for Virtual Population Analysis:

\[
C_t \times N_{t+1} = \left[\frac{F_t}{Z}\right] \times \left( e^{Z-1} \right)
\]

Given values of the catch \( C_t \), and an estimate of the natural mortality \( M \), the equation can be used to estimate retroactively the size of the past cohorts, if an estimate of \( N_{t+1} \) is available from which to start the computation. Estimates of \( N_{t+1} \) (expressing the last population size a cohort had before it became extinct) are called ‘terminal population’ \( N_t \).

Values of \( N_t \) can be obtained from:

\[
N_t = Z_t \times C_t / F_t \tag{13}
\]

Where \( C_t \) is the terminal catch (i.e., the last catch taken from a cohort before it went extinct) and \( F_t \) is the terminal fishing mortality (i.e., the fishing pressure that generated \( C_t \)).

A VPA starts with an initial guess of \( F_t \) and then calculates backwards with the known catches and natural mortality rate (Figure 9).

The major aim of cohort analysis is to estimate the fishing mortality \( F \) over the different length classes. The basics on how to do it are well explained by Sparre and Venema (1992, 1998) and only summarized here.

The basic formulae for a length based cohort analysis are:

\[
N(L1) = \left\lbrack N(L2) \times e^{\left(\frac{M \times dt}{2}\right)} \right\rbrack + C(L1, L2) \times e^{\left(\frac{M \times dt}{2}\right)}
\]

and

\[
C(L1, L2) = N(L1) \times e^{\left[1 - e^{-\frac{M \times dt}{2}}\right]}
\]

(14)

(15)

Where

\( C(L1, L2) \) the number of fish caught of length between \( L1 \) and \( L2 \)

\( N(L1) \) the number of fish that attain length \( L1 \)

\( dt \) time needed for the fish to grow through length \( L1 \) to length \( L2 \)

\( M \) natural mortality during time \( dt \)

\( F \) fishing mortality during time \( dt \)

\( Z \) total mortality during time \( dt \)

In a cohort analyses with non-seasonal growth

\[
e^{\left(\frac{M \times dt}{2}\right)} = \left[\frac{L_\infty - L1}{L_\infty - L2}\right]^{\frac{M}{2K}}
\]

and

\[
dt = \left(\frac{1}{K}\right) \ln \left[\frac{L_\infty - L1}{L_\infty - L2}\right]
\]

(9)

(10)

(11)

(12)

(13)

(14)

(15)

Again with seasonal growth the last formulae will give incorrect results. However, \( dt \) can be calculated as:

\[
dt = \text{age} \ L2 - \text{age} \ L1 \text{ and can be solved with the LOOKUP function. Then we can use the basic formulae directly.}
\]

In Figure 10 an example of a cohort analysis in a spreadsheet for Puntius sophore with a natural mortality of \( M = 1.168 \text{ year}^{-1} \) (all other parameters being the same as those in the previous examples) is presented. The cell formulae are presented in Figure 11.
In Figure 12 the fishing mortality (F) for the different length classes, calculated with non-seasonal and seasonal growth, is presented. The comparison indicates that the traditional non-seasonal method underestimates F during the period of slow growth, which in our example is occurring at a length of 8-9 cm. The main reason is that the fish stay for a long period in this length class, while fishing continues. The difference seems to be subtle, but the practical consequences can be large. For example, in Bangladesh, growth slows during the dry season (December-January). During this period the floodplains are small as they are drying up and the fishing effort is high. Underestimating the fishing mortality during this period will have serious consequences if the estimates are used in the Thompson and Bell models for the comparison of fisheries management options.

In this respect, it is important to notice that the LOOKUP function can be used in a similar way to calculate $dt$ for seasonal growth in the Thompson and Bell models and in the seasonal version of the Yield per Recruit Analyses (Sparre 1991).

**Limitation of the Proposed Method**

The proposed method provides convenient results but has some limitations. First of all, the time of recruitment has to be known. This is a minor limitation as, in most cases, the major spawning month is known. Secondly, the used seasonal version of the VBGF of Somers (1988) has exactly one zero growth rate per year when $C = 1$, which means that for each length there will always be one value for age. Using this version of the VBGF will do for most tropical fisheries, where prolonged periods of zero growth are an exception. The method cannot be applied if the...
A similar problem arises if there are two cohorts per year, which is often the case in penaeid shrimps. There again, there is no one-to-one correspondence between age and length (Sparre 1990). In this case, the only solution is to slice the cohorts (Sparre and Venema 1998) and apply a VPA with pseudo cohorts (Pauly et al. 1987; Gayanilo and Pauly 1997).

The Spreadsheets

The different spreadsheets can be downloaded from our website www.nefisco.org/Training.htm

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